Glossary

Thermal Model of Thermal Network Method

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1 Introduction

This Glossary describes the Thermal Model used in the thermal network method, which is an approach to thermal design. With a focus on similarities between heat conduction and electrical conduction, the thermal network method relies on the substitution of a heat transfer route by a thermal circuit consisting of thermal resistance and thermal capacity as shown in Table 1, from which temperature and heat flow can be calculated for thermal design.

Table 1 Relationships between thermal conduction and electrical conduction

Thermal	Unit		Electrical	Unit
Temperature	K	→	Voltage	V
Flow	W(=J/s)	→	Current	A
Resistance	K/W	→	Resistance	Ω(=V/A)
Capacity	J/K	→	Capacitance	F(=A・s/V)

Heat transfer can take place in three different modes: "heat conduction" in a solid body, "thermal radiation" by means of electromagnetic waves, and "heat convention" from a solid to a fluid. Heat itself is a kind of kinetic energy of molecules and atoms. To create a thermal model, it is necessary to understand the materials used in electronic components and determine how they physically behave at the atom/electron scale. On the other hand, product design only requires a simplified model for rough estimation of heating temperature, not an accurate model based on a strict physics theory. Simply creating an appropriate circuit model with materials and geometry taken into account will enable the quick calculation of heating temperature using an electronic circuit simulator. How to configure a circuit model is the key to obtain results approximating actual temperatures. The following explains the knowhow with expressions that are as accurate as possible while giving higher priority to introducing of the model in an easy-to-understand way.

2 Temperature, Movement of Atoms, and Thermal Capacity

Assume that an atom having a mass m moves at a velocity v in a gas at a temperature T. According to the principle of equipartition, the kinetic energy E of the atom can be expressed by Equation (1):

$$E=\frac{1}{2}mv^2=\frac{3}{2}kT \tag{1}$$

where k is a physical constant called Boltzmann's constant of 1.38 x 10^{-23} J/K. If T = 300 K, the energy of the single atom at room temperature (26.85°C) can be expressed by Equation (2):

$$E=\frac{3}{2}kT=6.21\times10^{-21} \tag{2}$$

For copper, for instance, the mass m of the atom is 1.06 x 10^{-25} kg. From the energy value expressed by Equation (2), the average velocity v of the atom is 342 m/s as expressed by Equation (3):

$$v=\sqrt{\frac{2E}{m}}=342 \tag{3}$$

Next, assume there is a Japanese 10-yen coin at room temperature (Fig. 1). The copper atoms constituting the coin have kinetic energy equivalent to the average velocity indicated by Equation (3). These copper atoms are bound together with adjacent copper atoms by metal-to-metal bonding and can only vibrate within a limited range. In this case, it can be said that the kinetic energy proportional to the temperature has been accumulated in the 10-yen coin. In other words, the 10-yen coin serves as a capacity to store the heat. The total kinetic energy U of the atoms is proportional to the temperature T and can be expressed by Equation (4). The proportionality constant C represents thermal capacity and is defined as the amount of heat required to increase the temperature of an object by 1 K. The unit is [J/K].

$$U = CT \tag{4}$$

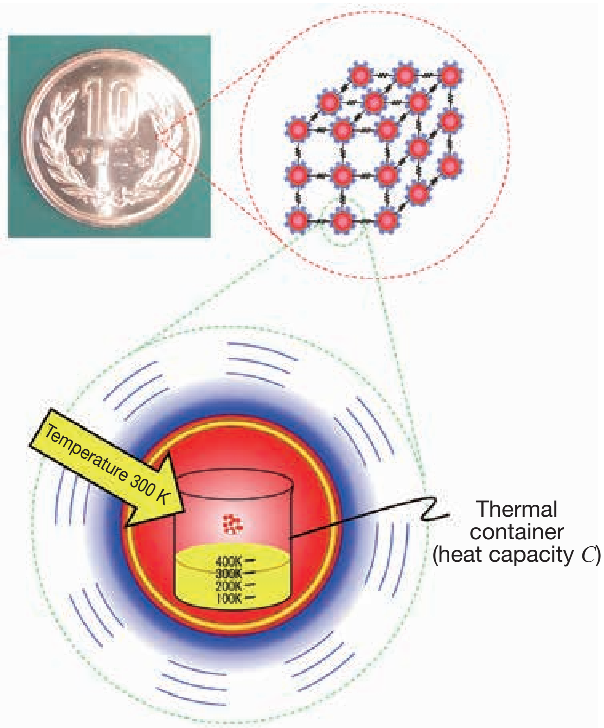

Fig. 1 Movement of copper atoms constituting a 10-yen coin

3 Mutual Vibration and Thermal Resistance of Atoms

Next, consider the force to bind atoms of a solid body together (Fig. 2).

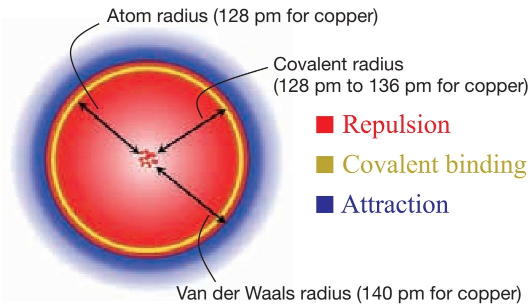

Fig. 2 Distance from the center of the atom and forces applied

Atoms (ions not included here) a certain distance away from each other are held together by forces of attraction (the Van der Waals force). When these atoms come closer to each other, the forces of attraction are replaced by the forces of repulsion at a certain distance between them (the Van der Waals distance). When they come still closer to each other, they share electrons of neighboring atoms to have covalent bonding, keeping their distance at a fixed length (covalent distance). If they are forced to be even closer to each other, they strongly repel against each other in the manner of a rigid body (atomic radius). These forces exerted among the atoms are used to transfer the vibration energy among the neighboring atoms (Fig. 3).

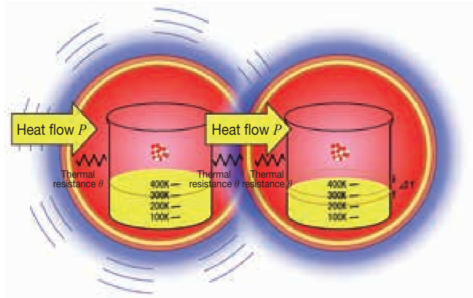

Fig. 3 Distance from the center of the atom and forces applied

Since the vibration energy is gradually transferred with the forces of attraction and repulsion, there arises a time difference in propagation, hindering the heat conduction. Assuming this is thermal resistance θ, the relationship between the heat flow P and temperature change ΔT caused by the flow can be expressed by Equation (5):

$$\Delta T = \theta P \tag{5}$$

This relationship may be substituted by an electric circuit and can be expressed by a multi-stage filtering circuit with the capacitance stated above, as shown in Fig. 4:

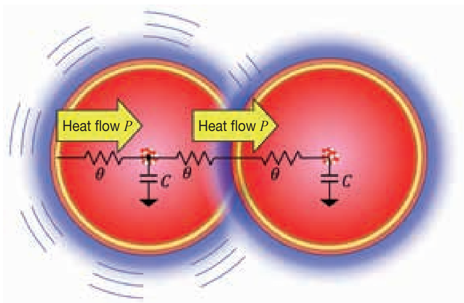

Fig. 4 Heat conduction model of atoms

Fig. 5 shows another model of a greater number of atoms, expanded from the two-atom model shown above, representing how the heat flow P sequentially propagates to neighboring atoms:

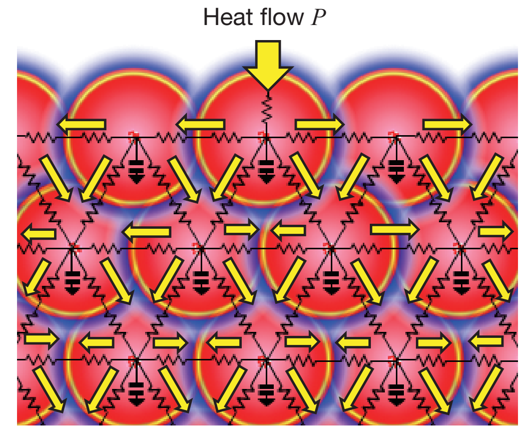

Fig. 5 Heat propagation model of multiple atoms

4 Modeling of Heat Conduction Within a Solid Body

A heat conduction model of a solid body may be created by joining a number of heat conduction models of atoms. For such a model, however, the calculation load is very high. For this reason, it should be converted into a simpler model with similar behavior.

Fig. 6 shows an example of such models. When an atom receives the heat flow P, it transfers the heat to its neighboring atom. This operation is similar to a bucket brigade. If a single atom is replaced by a thermal circuit, these models can be substituted by a T-type low pass filter (LPF). Several LPFs can be combined to create a thermal network.

The first resistance to receive the heat flow P may be omitted in a case of calculating from the temperature T_1 of the first atom. Two resistances that bind two atoms may be put together as a single resistance.

Furthermore, the n-stage LPFs for atoms can be approximated by a primary LPF if the conditions are met.

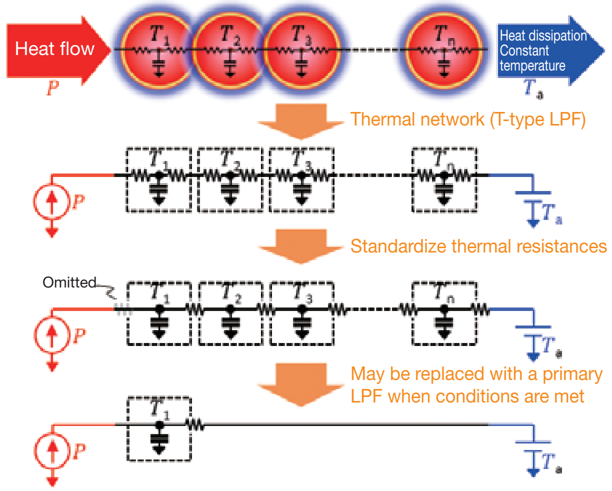

Fig. 6 Simplified heat conduction model of solids

Assuming that the input heat flow is P, the input temperature T_1, and the output temperature T_a, Equation (6) for a primary LPF consisting of resistance θ and capacity C can be obtained:

$$\frac{T_1 - T_a}{P} = \frac{1}{sC + \frac{1}{\theta}} = \frac{\theta}{sC\theta + 1} \tag{6}$$

In the steady state (s → 0), the effect of the thermal capacity C is negligible. The calculation can be made by Equation (7):

$$T1 - Ta = \theta P \tag{7}$$

A tip for modeling is to replace atoms with an LPF of an appropriate order based on the materials and geometry of the elements.

5 Modeling of Conductive Heat Transfer with Free Electrons

Heat conduction in a solid body involves heat propagation with lattice vibration as well as the action of free electrons if the body is metal. Metal is popularly used as heat dissipating materials such as heat sink and substrates and is an indispensable element of the heat conduction model.

Fig. 7 shows a metallic square pole of a cross-sectional area S, a length l and a thermal conductivity λ. The thermal resistance θ on both sides of the metallic pole is defined by Equation (8). The temperature change can be expressed by Equation (9) using the thermal resistance and heat flow P:

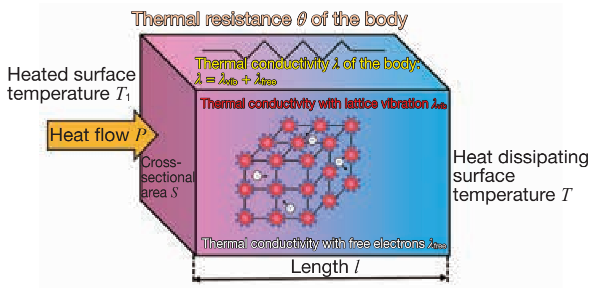

Fig. 7 Thermal conduction model of metal

$$\theta = \frac{l}{S\lambda} \tag{8}$$

$$T_1 - T_a = \theta P = \frac{l}{S\lambda} P \tag{9}$$

The electrical and thermal conductivities of metal show a coefficient of correlation as high as 99.6% as shown in

Fig. 8, known as the Wiedemann-Franz law. The law implies the high contribution of free electrons to the thermal conductivity of metal.

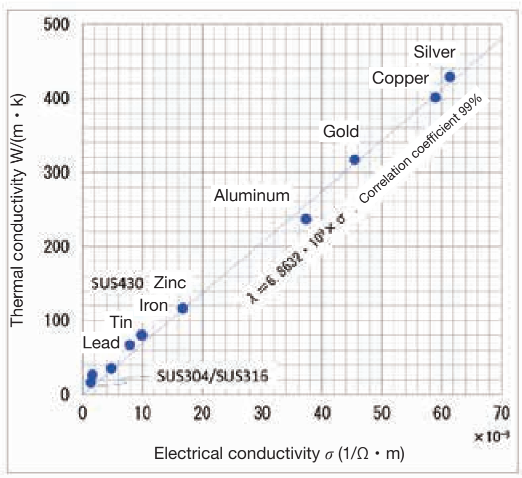

Fig. 8 Relationship between thermal and electrical conductivities of metal

6 Modeling of Thermal Radiation

Thermal radiation is a phenomenon in which thermal energy is emitted from an object as electromagnetic waves. According to the Stefan-Boltzmann law, heat flow proportional to the temperature to the 4th power is released. Since thermal radiation takes place on the surfaces of a solid body, this is an important mode of heat dissipation on a surface exposed to the atmosphere.

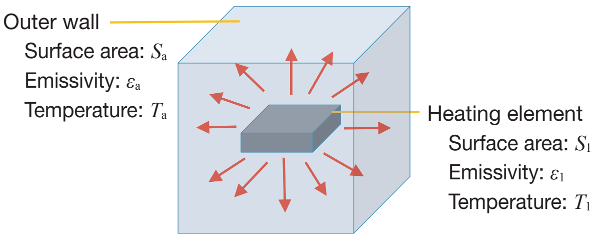

Fig. 9 Illustration of thermal radiation

As shown in Fig. 9, the thermal flow radiated from the outer wall of a surface temperature S_a, a temperature T_a, and an emissivity ε_a of a component having a surface area S_1, a surface temperature T_1, and an emissivity ε_1 to the atmosphere can be expressed by Equation (10). The emissivity ε_1, ε_a is a correction factor between 0 and 1 and is decided by the material and roughness of the surface of the solid body. σ is the Stefan-Boltzmann constant and takes a value of 5.67 x 10^{-8} W/(m^2 • K^4). If $S_a \gg S_1$, that is, heat is transferred to a distance, it can be approximated by Equation (11) and the surface temperature of the solid body can be determined by Equation (12).

This relationship can be substituted by an electric circuit with a constant voltage source and resistances as shown in Fig. 10:

$$P = \frac{\sigma}{\dfrac{1}{\varepsilon_1} + \dfrac{S_1}{S_a}\left(\dfrac{1}{\varepsilon_a} - 1\right)} S_1 (T_1^4 - T_a^4) \tag{10}$$

$$P = \varepsilon_1 \sigma S_1 (T_1^4 - T_a^4) \tag{11}$$

$$T_1 - T_a = \theta P = \frac{1}{\varepsilon_1 \sigma S_1 (T_1^2 + T_a^2)(T_1 + T_a)} P \tag{12}$$

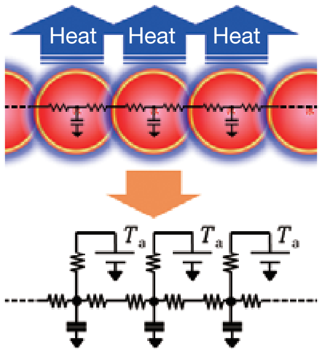

Fig. 10 Thermal radiation heat transfer model

7 Modeling of Convective Heat Transfer

Convective heat transfer involves the transfer of heat from one place at a higher temperature to another at a lower temperature. For an electronic component that generates heat, the heat should be released to the atmosphere to cool the heated component. This heat dissipation is convection.

Since most electronic components are solid, the following discusses heat transfer from a solid to a gas.

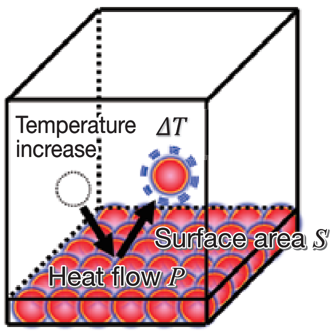

Fig. 11 Illustration of convective heat transfer

On the surface of a heat-generating solid body the atoms are thermally vibrating. When the surface of the solid body is exposed to a gas, the gas receives the thermal vibration, thereby increasing in temperature. After giving off the energy from the surface, the solid body has less thermal vibration, thereby decreasing in temperature. This process takes place repeatedly with a number of molecules to achieve cooling (Fig. 11).

Assuming that the surface temperature of the solid body is T_1, the atmosphere temperature T_a, the surface area of the solid body S, and the heat transfer coefficient h, the heat flow can be expressed by Equation (14) according to Newton's law of cooling (Fig. 59):

$$P = hS(T_1 - T_a) \tag{13}$$

$$T_1 - T_a = \theta P = \frac{1}{hS} P \tag{14}$$

The heat transfer coefficient refers to the amount of heat that is transferred per unit of time in unit of area when the difference in temperature between the surface of a body and its surrounding atmosphere is 1 K. Practically, 5 to 10 W/(k・m^2) is generally used. A lower value may be used in an enclosed space.

As in the case of thermal radiation, the convective heat transfer model can be substituted by an electric circuit with a constant voltage source and the resistances shown in Fig. 10.

References

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