Glossary

Compensators in Control Systems

Refer to “Control Technologies for In-Vehicle Electric Actuators” (page 8)

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1 What is a Compensator?

1.1 Role of Compensators

The compensator [Note 1)] in a control system refers to a component that generates manipulated variables (control input) to attain a desirable response (control output) from the system.

Note 1) This document uses the term “compensator”, not the term “controller” which may sometimes be used.

As an example, a case is considered in which a vehicle is applied with a driving force u (control input) and controlled so that its velocity v (control output) is maintained at a fixed target velocity v_r (setting value) (Fig. 1). The vehicle is affected by disturbances d such as aerodynamic drag, friction against the road surface, and forces caused by gravity on slopes. It is assumed here that disturbances d are almost constant.

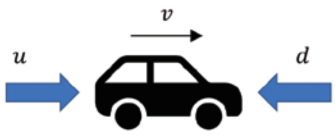

Fig. 1 Vehicle velocity control

Then, a control system is configured as shown in Fig. 2:

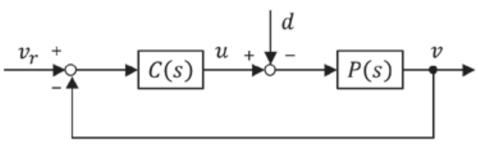

Fig. 2 Vehicle velocity control system (1)

where

$P(s)$: Transfer function of controlled object (transfer function from the force applied to the vehicle u-d to the velocity v)

$C(s)$: Transfer function of compensator

s: Laplace operator (or differential operator)

For the control system in Fig. 1, the velocity v of the vehicle can be expressed by the equation below:

$$v=\frac{C(s)P(s)}{1+C(s)P(s)}v_r-\frac{P(s)}{1+C(s)P(s)}d \tag{1}$$

As time elapses ($s\to 0$), the vehicle velocity v can be expressed by the equation below:

$$v=\frac{C(0)P(0)}{1+C(0)P(0)}v_r-\frac{P(0)}{1+C(0)P(0)}d$$
$$=\frac{1}{\frac{1}{C(0)P(0)}+1}v_r-\frac{\frac{1}{C(0)}}{\frac{1}{C(0)P(0)}+1}d \tag{2}$$

When the compensator $C(s)$ is designed so that $C(0)$ is sufficiently large, $1/C(0)$ in Eq. (2) is almost zero (0). The vehicle velocity v is almost $v = v_r$ accordingly. Designing the compensator $C(s)$ in this way will achieve one of the control objectives, that is, “to keep the velocity close to the target value”. Behind the control objectives lie a wide range of needs including cost-effectiveness, safety, and comfort. The role to be played by the compensator in order to satisfy these needs will be described in Chapter 3.

1.2 Types of Compensators and Example of Control System Characteristics

Typical compensators include proportional, differential, integral, phase lead, and phase lag compensators. A detailed description of these compensators is omitted here.

To achieve one of the control objectives “to keep the velocity close to the target value” stated in the previous section, a proportional and integral compensator will be sufficient, provided that the transfer function of the controlled object can be expressed by the equation below:

$$P(s)=\frac{1}{Ms} \tag{3}$$

where M indicates the weight [kg] of the vehicle. Eq. (4) represents an ideal vehicle that is applied with a control input u without delay. The transfer function of the proportional and integral compensator can be expressed by the equation below:

$$C(s) = K_P + \frac{K_I}{s} = \frac{K_P s + K_I}{s} \qquad (4)$$

where K_P and K_I are proportional and integral gains, respectively. When Eqs. (3) and (4) are substituted in Eq. (1), the vehicle velocity v can be expressed by the equation below:

$$\begin{aligned} v &= \frac{C(s)P(s)}{1+C(s)P(s)} v_r - \frac{P(s)}{1+C(s)P(s)} d \\ &= \frac{\frac{K_P}{M}s + \frac{K_I}{M}}{s^2 + \frac{K_P}{M}s + \frac{K_I}{M}} v_r - \frac{\frac{1}{M}s}{s^2 + \frac{K_P}{M}s + \frac{K_I}{M}} d \qquad (5) \\ &= \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} v_r - \frac{\frac{1}{M}s}{s^2 + 2\zeta\omega_n s + \omega_n^2} d \end{aligned}$$

where

$$\omega_n = \sqrt{K_I/M},\ \zeta = K_P\sqrt{1/(K_I M)}/2$$

Setting K_P and K_I to achieve $0 < \omega_n$ and $0 < \zeta$ will make the control system stable. The equation also implies that, as time elapses ($s \rightarrow 0$), $v = v_r$ will hold.

1.3 How to Configure Compensators

There are two types of compensator configuration: analog and digital. An analog compensator uses resistors, capacitors, operational amplifiers, and other elements. The example shown in the previous section assumes control with an analog compensator. The transfer function of an analog compensator is represented using the Laplace operator s (or differential operator). A digital compensator is configured using a logic circuit or digital computer. Digital compensators have the advantage that complex compensators can be configured using software, thereby enabling the user to easily modify parameter settings. Today, many control systems for in-vehicle actuators use digital compensators because the performance of microprocessors has been improved. The transfer function of a digital compensator is represented using the z and δ operators.

2 Parametrization of Stabilizing Compensators

Parametrization of stabilizing compensators was proposed by Youla and others during the 1970s [1]. This may be called "Youla parametrization". Studies on robust control theory became active toward the 1990s and many books explaining how to apply Youla parametrization to actual control scenes [2] were published. A stabilizing compensator design with parametrization should represent "how the compensator should be" derived from a model of the controlled object and the desirable characteristics of the control system, instead of "first selecting the type of the compensator", such as proportional or integral as introduced in section 1.2, before designing. In other words, the designer can find a compensator satisfying the desirable characteristics of the control system even if the controlled object is quite complex. Still, a more-or-less accurate model of the controlled object (a mathematical model such as transfer function) will be needed. In some cases, the achievable characteristics of the control system may be limited depending on the characteristics of the controlled object. For your information, the proportional and integral compensator stated in section 1.2 can also be derived from the parametrization of a stabilizing compensator.

3 Two-degree-of-freedom Control System

The control system described in Chapter 1 may have other control objectives than just "to keep the velocity close to the target value", attributable to various demands including "[1] to reduce the fuel (or electricity for electric vehicles) consumption during acceleration (cost-effectiveness)", "[2] to reduce the shock felt by passengers during acceleration or deceleration (ride comfort)", and "[3] to minimize speed variations during driving on slopes (safety)". Demands [1] and [2] mainly relate to target tracking and [3] relates to disturbance response. With a focus on Eq. (1) in section 1.1, target tracking (the first term on the right-hand side) and disturbance response (the second term on the right-hand side) are both decided by the compensator $C(s)$. This means that, in some cases, demands [1] & [2] and demand [3] can hardly all be satisfied at the same time. Then, another control system should be considered as shown in Fig. 3:

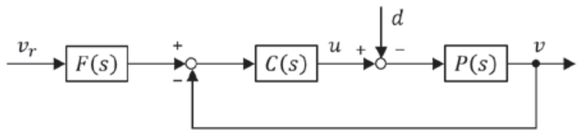

Fig. 3 Vehicle velocity control system (2)

The velocity v of the vehicle with the control system in Fig. 3 can be expressed by the equation below:

$$v = \frac{F(s)C(s)P(s)}{1+C(s)P(s)} v_r - \frac{P(s)}{1+C(s)P(s)} d \qquad (6)$$

With an additional compensator $F(s)$, target tracking (the first term on the right-hand side) and disturbance response (the second term on the right-hand side) can be set separately. That is, two-degree-of-freedom design is now available. In this sense, the control system in Fig. 3 is called a two-degree-of-freedom control system.

References

1) D.C. Youla, Hamid A. Jabr, J.J. Bongiorno Jr: Modern Wiener-Hopf Design of Optimal Controllers - Part II: The Multivariable Case, IEEE Trans. Auto Control, AC-21, No. 3, pp. 319-338, (1976).
2) MAEDA, SUGIE: System Control Theory for Advanced Control, Asakura Publishing, (1990).