Technology Explanation

Control Technologies for In-Vehicle Electric Actuators

MATSUMOTO Daisuke, ENOMOTO Daisuke, KENJO Kazunori, FUKUSHI Natsuru

Abstract

KYB develops and manufactures various in-vehicle actuators including electric power steering (EPS). The control specifications of the actuator differ depending on the function, but it is a common issue to achieve a system that operates stably and satisfies the required functions and performance. In addition, when controlling the actuator, the control system may become unstable depending on the design of the control compensator, so it is necessary to design the compensator after grasping the characteristics of the actuator to be controlled.

In this report, we explain the compensator design method for the steering assist control of EPS for all terrain vehicles and utility task vehicles and for steering angle control with an eye toward future application in automatic steering, as well as the implementation method of this design method. This report also describes the future outlook.

1 Introduction

Compared with electric power steering (EPS) for passenger vehicles, EPS for all-terrain and utility task vehicles, particularly for off-road vehicles, is required to deliver more highly responsive steering assist and kickback control performance. To satisfy these performance requirements, it is a precondition that the EPS hardware (including the motor described later and its driving circuit, gear assembly, etc.) has been properly designed. In addition, it is necessary to maximize the hardware capabilities by using software (i.e., with a proper control methodology). Specifically, the open-loop gain of the control system (a gain of the loop transfer function) must be set to a level as large as possible within the frequency ranges in which the steering assist and kickback control performance are required. However, care must be exercised in setting the open-loop gain because just using a high open-loop gain would reduce the gain and phase margins (hereinafter "stability margin"), resulting in an unstable control system in some cases. As an approach to ensure a stability margin while using a high open-loop gain (hereinafter "stabilization"), a phase compensator [Note 1)] has been used. For stabilization using this approach, the phase compensator is designed with a focus on the gain and phase information of the frequency characteristics of the loop transfer function of the control system (called a "non-parametric model"). It is relatively easy to implement a phase compensator, but designers need to adjust design parameters through trial and error while carefully observing the frequency characteristics. This means that such designers need to have a certain level of experience and knack. As described later, the compensator is assumed to be redesigned in phases during the development stage. For higher efficiency in development, it is desirable to automate the compensator development processes, including from design to implementation, to some extent. However, design tool automation can hardly be achieved by conventional approaches.

Note 1) The compensator in a control system is a computing unit intended to generate control input to impart desirable characteristics to the system. A phase compensator is designed with a focus on the gain and phase of the loop transfer function of the control system.

On the other hand, robust control theory, typified by the H_∞ control theory, involves approaches that use the transfer function and state equation of the controlled object (called a "parametric model") to design the compensator through inverse operation from the desirable characteristics (target tracking and disturbance response) of the control system. In these approaches, the controlled object, the characteristics of the control system, and disturbances are represented by parametric models and the related various equations are solved to determine the compensator. This means that design tools can relatively easily be automated, delivering the benefit that compensators can be designed more efficiently. These design approaches may be effective to catch up with the rapid development specific to the all-terrain and utility task vehicles market.

In general, compensators designed based on robust control theory tend to be more complicated (with higher degree) than phase compensators. However, in-vehicle microprocessors mounted with floating point units (FPUs) have been generally introduced in recent years to allow implementation of compensators of relatively high degree.

This report explains how to design steering assist and steering angle control compensators for EPS based on the parametrization of stabilizing compensators [3], which is one of the basic theories of robust control.

2 EPS Systems

2.1 Components of EPS System

Fig. 1 shows the components of the EPS for all-terrain and utility task vehicles.

The electromechanical brushless motor shown in the Figure (hereinafter "the power pack") is a brushless motor integrated with a controller (hereinafter "the motor") as the name implies. This component performs computations for all controls including steering assist control.

The torque angle sensor (TAS) is another component that detects the torque and angle of the steering wheel operated by the driver. The steering assist control uses torque values detected by the TAS as described later. The steering angle control uses angle values detected by the TAS to set the origin of the steering angle that is calculated from the motor rotation angle.

The gear assembly has a mechanism by which the torque generated by the motor is amplified by a worm reducer and this rotary motion is converted into a linear motion by a rack and pinion device. This component transfers the steering torque produced by the driver and the steering assist torque generated by the motor to the vehicle's wheels as a cornering force.

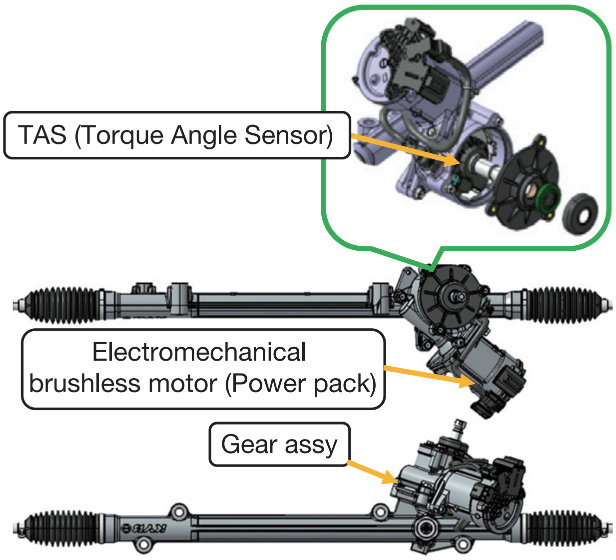


Fig. 1 EPS components for all-terrain and utility task vehicles

2.2 Block Diagram of EPS System

Fig. 2 is a block diagram of the EPS system.

The controller has three major functions; ① A monitoring function to detect any abnormality of the CPU, driving circuit and sensors, and perform fail-safe processing, ② A communication function to communicate with the vehicle and other sub systems via a CAN network, and ③ Control functions including motor vector control, steering assist control, and steering angle control.

With a focus on the steering assist and steering angle controls, the next chapter describes how to design compensators for these controls.

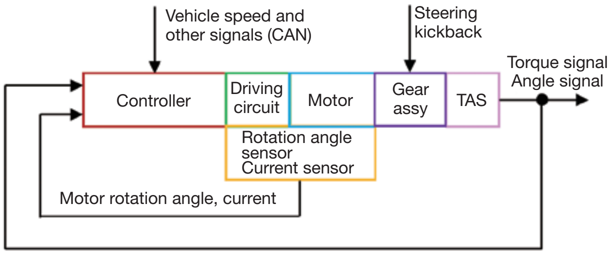


Fig. 2 Block diagram of EPS system

3 Overview of Compensator Design

3.1 Components of Control System

Fig. 3 shows an overview of the control system (the controlled object + compensator) with a focus on the steering assist control or steering angle control.

The control input to this control system is from motor current commands. The control output from the system is steering angle or torque values from the TAS (the steering angle is calculated from the motor rotation angle). The load torque is attributable to the driver's steering and kickback transferred to the motor via the gear assembly and the friction torque of the gear assembly.

In the Figure, the area enclosed by the red broken line represents the controlled object, which is a mixed system of hardware and software. The former includes the motor, the driving circuit, the controller, and the gear assembly, and the latter is for motor vector control and steering angle computation.

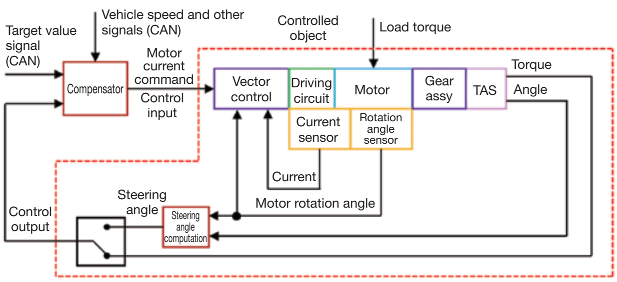


Fig. 3 Overview of control system

3.2 Compensator Design Policy

Within the controlled object shown in Fig. 3 in the previous section, the hardware including the driving circuit, motor, and gear assembly behaves electrically or mechanically in real time. With these physical phenomena, the hardware constitutes a continuous time system. On the other hand, the software is a discrete time system [Note 2] because vector control and steering angle computation are performed by digital computers. Therefore, the actual controlled object is a mixed system of continuous and discrete time systems. The control input to and output from the controlled object are updated or sampled at each of their own control frequencies. That is, the controlled object ranging from the control input to the control output can be represented by a discrete time model [Note 3]. The compensator can also be represented by a discrete time model because its computation is performed by digital computers.

Note 2) A system whose behavior is defined with discrete time (hereinafter "sampling time").

Note 3) A discrete time model is a difference equation or transfer function representation of the behavior of the controlled object under the control of a digital computer at a sampling time or the behavior of a discrete time system.

Conventionally, the set of the controlled object and compensator has been considered as a continuous time system. In this approach, the compensator is designed according to continuous time and then discretized using, for example, a bilinear expression before implementation. This approach has the problem that design stability cannot be ensured in actual systems. One major factor is that a compensator that has been discretized with a bilinear expression, for instance, cannot necessarily become a stabilizing compensator for the discrete time system (i.e., the stability margin for the discrete time system cannot be guaranteed) (Challenge ①). Another major factor relates to modeling error of the controlled object (discrepancy between the model and the actual system). The system cannot be stable if the designed compensator was not a stabilizing compensator for the actual system in the first place (Challenge ②). To address Challenge ①, it is useful to represent the controlled object as a discrete time model and design the compensator according to the discrete time system. For Challenge ②, it is effective to use the frequency response method or the method of identification [4)] using maximum-length sequence signals to determine a model for designing the compensator (hereinafter a "compensator design model"). Then, we have decided to introduce the approach of representing the controlled object in a discrete time model and designing the compensator in a discrete time system. We also identify the system to be controlled upon completion of the hardware fabrication and design the compensator using the identified compensator design model. We use δ (delta) operator [5)] as a discrete time operator representing the controlled object and compensator model. The δ operator can be expressed as $\delta = (z - 1)/T_c$ (where z is the z operator and T_c is control frequency). A major advantage of using the δ operator is that relatively high accuracy can be obtained with compensator computation with a low number of bits. For more information, see References [4), 5)].

3.3 Block Diagram of Control System

The controlled object included in the control system shown in Fig. 3 can be approximately represented by a block diagram as shown in Fig. 4:

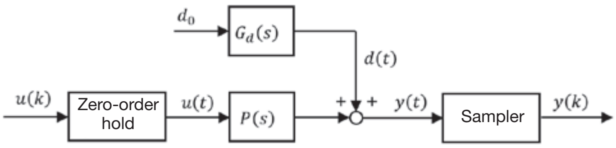


Fig. 4 Block diagram of controlled object (continuous time system)

$P(s)$ in Fig. 4 is the continuous time model (transfer function) of the controlled object. k is an integer not less than 0 (zero) used to indicate the sampling time [$t_s(k) = kT_c$ (k = 0, 1, ...)]. $u(k)$ is the output from the compensator, namely, control input (motor current command). $u(t)$ is $u(k)$ held every "time" $t_s(k)$. $y(k)$ is the control output that is $y(t)$ sampled every "time" $t_s(k)$. Control output $y(k)$ will be a TAS torque signal for steering assist control or a steering angle for steering angle control. $d(t)$ is the disturbance attributable to the load torque shown in Fig. 3 and the rotation of the steering wheel operated by the driver. d_0 is a constant. $G_d(s)$ is the transfer function representing a disturbance generator. Note that $u(t)$ and $u(k)$ are signals that are different from $y(t)$ and $y(k)$ signals, but for convenience these are not differentiated by symbols.

The block diagram of Fig. 4 may be represented by a discrete time system as shown in Fig. 5. $P(\delta)$ and $G_d(\delta)$ are transfer functions (discrete time models) obtained by zero-order holding and discretization $P(s)$ and G_d (s) with a sampler, respectively. Note that $P(s)$ and $P(\delta)$ are transfer functions that are different from G_d (s) and $G_d(\delta)$ transfer functions, but for convenience these are also not differentiated by symbols.

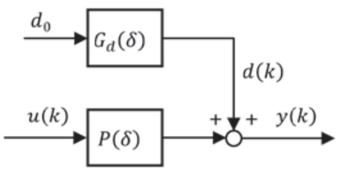


Fig. 5 Block diagram of controlled object (discrete time system)

Thus, the control system in Fig. 3 can be represented using the transfer function model $P(\delta)$ of the controlled object and the transfer function model $C(\delta)$ of the compensator as shown in Fig. 6. In Fig. 6, $r(k)$ is the target steering angle signal. r_0 is a constant. $G_r(\delta)$ is the transfer function representing the target value signal generator. This report describes how to design a compensator $C(\delta)$ for the control system in Fig. 6. Note that $C(\delta)$ is assumed to be a two-degree-of-freedom controller as follows:

$$C(\delta) = [C_r(\delta) \quad C_y(\delta)] \tag{1-1}$$

$$u(k) = C_r(\delta)r(k) - C_y(\delta)y(k) \tag{1-2}$$

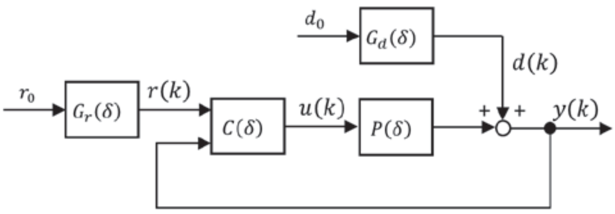


Fig. 6 Block diagram of control system

3.4 Compensator Design Flow

Fig. 7 shows the compensator design flow as a V-shaped process diagram. This section describes what to do in each of the design processes (the flow to the left in Fig. 7).

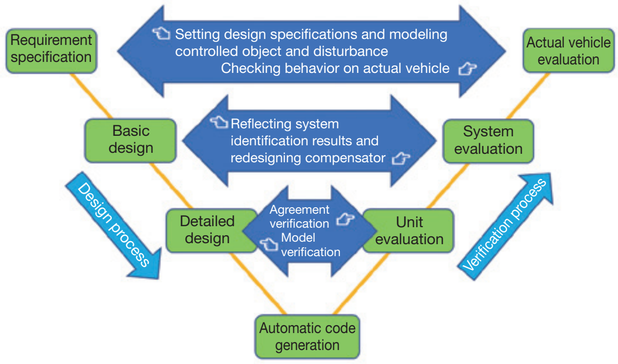


Fig. 7 V-shaped process of compensator design

3.4.1 Requirement Specification

The following describes what to do in the Requirement Specification stage.

(1) Setting design specifications

For steering assist control, an assist map where motor current commands are mapped against TAS torque signals is used to set the assist torque. A higher assist torque means a larger open-loop gain. A major requirement for the design specifications is that the required assist torque can be generated stably (i.e., without issuing harmful vibration). In other words, the control system will remain stable even with the maximum assumed open-loop gain.

For steering angle control, the steering angle will follow the target steering angle signal $r(k)$ in Fig. 6. Major design specifications include, for example, delay time, rise time, setting time, overshoot, and steady-state deviation to $r(k)$ that may change stepwise or in ramp rate.

(2) Modeling controlled object and disturbances

Controlled object models can roughly be divided into two types. One is a high-precision simulation model [6] that can estimate in relative detail the behavior of the actual system (hereinafter "the simulation model"). The other is a simplified model for designing compensators (the aforementioned compensator design model). A compensator design model can be derived from a differential equation that has linearized and simplified the controlled object. A compensator design model can also be derived through identification of a system using the simulation model or actual machine. The identification of systems using an actual machine will be described later. The simulation model can be created and implemented mainly by using tools such as MATLAB®/Simulink® and SimulationX®.

The disturbances that may be applied to the controlled object include; ① Rotation of the steering wheel operated by the driver (for steering assist control), ② Load torque due to kickback, and ③ Friction torque of the gear assembly. For steering assist control, these disturbances can be represented by a transfer function model with an assumption of steering frequency as described later. For steering angle control in turn, the disturbances should be represented by a step or ramp function model based on the assumption that, with the driver's hands off the steering wheel, the steering angle can follow the steering angle command without steady-state deviation under disturbances ② and ③.

3.4.2 Basic Design

In the basic design stage, a compensator is designed using the compensator design model and the assumed disturbance model. The compensator can be expressed by a transfer function model $C(\delta)$ in this stage. The simulation model and the designed compensator are used to carry out simulation and verify the adequateness of the compensator during this stage. The compensator design will be described in detail in Chapter 4.

3.4.3 Detailed Design

In the detailed design stage, the compensator designed in the basic design stage is developed into an implementation model to be contained in the controller as software. This stage should verify that the compensator $C(\delta)$ derived in the basic design stage is equivalent to the implementation model (i.e., identical inputs will produce the same output).

3.4.4 Automatic Code Generation

Embedded Coder® is used to automatically generate codes from the implementation model.

4 Compensator Design

This chapter describes the details of the basic design of compensators for steering assist control and steering angle control. The compensator design for steering assist control is different from that for steering angle control in control output, design specifications, and disturbances. The two compensators can be applied with the same design approach.

4.1 Basic Design of Steering Assist Control Compensator

4.1.1 Deriving a Compensator Design Model

An EPS model is shown in Fig. 8:

The EPS shown in Fig. 8 is a single-pinion type. This model can also be applied to double-pinion or column

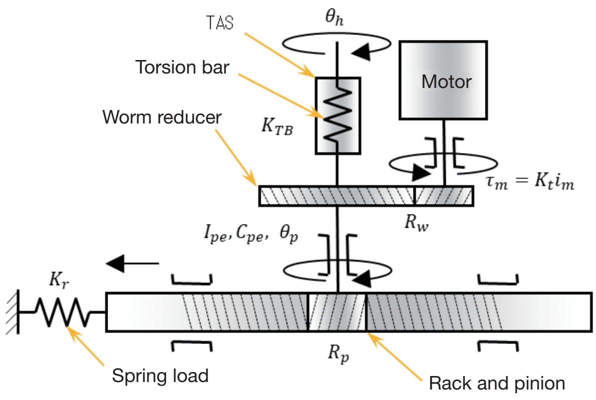


Fig. 8 EPS model

EPS. The symbols used in the figure indicate the following:

θ_h : Rotation angle of steering wheel [rad]
θ_p : Rotation angle of pinion [rad]
i_m : Motor current [A], τ_m: Motor torque [N·m]
K_t : Motor torque constant [N·m/A]
K_{TB} : Spring constant of torsion bar [N·m/rad]
R_w : Reduction ratio of worm reducer
I_{pe} : Equivalent inertia moment of pinion shaft [kg·m^2]
C_{pe} : Equivalent viscous resistance coefficient of pinion shaft [N·m/(rad/s)]
R_p : Specific stroke of rack and pinion [m/rad]
K_r : Spring constant of rack load [N·m/rad]

The TAS torque signal can be expressed by the equation below:

$$\tau_s = K_{TB}(\theta_h - \theta_p) \tag{2-1}$$

Assuming that i_m is equal to u (the actual current completely follows the motor current command) and that all the parts except the torsion bar are rigid bodies, the rotary motion of the pinion can be expressed by the differential equation below:

$$I_{pe}\ddot{\theta}_p + C_{pe}\dot{\theta}_p = R_w K_t i_m + K_{TB}(\theta_h - \theta_p) - K_r R_p^2 \theta_p \tag{2-2}$$

where

$i_m = u$

From Eqs. (2-1) and (2-2), the following state and output equations can be obtained:

$$\dot{x} = A_P x + B_P u + E_P \theta_h \tag{2-3}$$
$$y = C_P x + F_P \theta_h \tag{2-4}$$

where

$x = [\theta_p \quad \dot{\theta}_p]^T$: State variable, $y = -\tau_s$: Control output
u: Control input (motor current command)

$$A_P = \begin{bmatrix} 0 & 1 \\ -K_{pe}/I_{pe} & -C_{pe}/I_{pe} \end{bmatrix}, \; K_{pe} = K_{TB} + K_r R_p^2$$

$$B_P = \begin{bmatrix} 0 \\ R_w K_t / I_{pe} \end{bmatrix}, \; C_P = [K_{TB} \quad 0]$$

$$E_P = \begin{bmatrix} 0 \\ K_{TB}/I_{pe} \end{bmatrix}, \; F_P = -K_{TB}$$

In Eqs. (2-3) and (2-4), if θ_h is 0, the state and output equations for the discrete time model can be expressed by the equations below:

$$\delta x(k) = A_{P\delta} x(k) + B_{P\delta} u(k) \tag{3-1}$$
$$y(k) = C_{P\delta} x(k) \tag{3-2}$$

where

$A_{P\delta} = (A_{Pz} - I_2)/T_c$, I_2: 2 × 2 unit matrix
$B_{P\delta} = B_{Pz}/T_c$, $C_{P\delta} = C_{Pz} = C_P$
$A_{Pz} = e^{A_P T_c}$, $B_{Pz} = \int_0^{T_c} e^{A_P T_c} d\tau \, B_P$

A_{Pz}, B_{Pz} and C_{Pz} are a system/control matrix where Eq. (2-3) or (2-4) is discretized by the z operator each. These can be determined using the discretization method "zoh" using the MATLAB® function "c2dm". Eqs. (3-1) and (3-2) are called step invariant models[5] of Eqs. (2-3) and (2-4). When Eqs. (3-1) and (3-2) are selected for the compensator design model, the transfer function $P(\delta)$ in Figs. 5 and 6 can be expressed by the equation below:

$$P(\delta) = C_{P\delta}(\delta I_2 - A_{P\delta})^{-1} B_{P\delta} = \frac{n_P(\delta)}{d_P(\delta)} = \frac{n_{P1}\delta + n_{P0}}{\delta^2 + d_{P1}\delta + d_{P0}} \tag{3-3}$$

In the initial development stage where the detailed design of hardware has not been completed, Eqs. (3-1) to (3-3) should be used as compensator design models to design compensators. However, these models may need to be revised when specific hardware specifications are made available or when the hardware fabrication has been completed. With consideration given to these cases, the degree of the compensator design models (degree of the denominator polynomial $d_p(\delta)$) is assumed to be 'n' in the following discussion. Then, the transfer function $P(\delta)$ is expressed by the equation below:

$$P(\delta) = \frac{n_P(\delta)}{d_P(\delta)} = \frac{n_{Pn-1}\delta^{n-1} + \cdots + n_{P1}\delta + n_{P0}}{\delta^n + d_{Pn-1}\delta^{n-1} + \cdots + d_{P1}\delta + d_{P0}} \tag{3-4}$$

4.1.2 Setting Disturbance Models

This section discusses disturbance models of degree l that can be expressed by the equations below:

$$d = G_d(s) d_0 \tag{4-1}$$

$$G_d(s) = \frac{n_d(s)}{d_d(s)} = \frac{n_{dl-1}s^{l-1} + \cdots n_{d1}s + n_{d0}}{s^l + d_{dl-1}s^{l-1} + \cdots d_{d1}s + d_{d0}} \tag{4-2}$$

The state and output equations of Eq. (4-1) can be expressed by the equations below:

$$\dot{x} = A_d x + B_d d_0 \tag{4-3}$$
$$d = C_d x \tag{4-4}$$

where

$$A_d = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -d_{d0} & -d_{d1} & -d_{d2} & & -d_{dl-1} \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \; C_d = [n_{d0} \quad n_{d1} \quad n_{d2} \quad \cdots \quad n_{dl-1}]$$

The state and output equations of the discrete time model of disturbances can be expressed by the equations below:

$$\delta x(k) = A_{d\delta} x(k) + B_{d\delta} d_0 \quad (4\text{-}5)$$

$$d(k) = C_{d\delta} x(k) \quad (4\text{-}6)$$

where

$A_{d\delta} = (A_{dz} - I_l)/T_c$, I_l: $l \times l$ unit matrix

$B_{d\delta} = B_{dz}/T_c$, $C_{d\delta} = C_{dz}$

$A_{dz} = e^{A_d T_c}$, $B_{dz} = \int_0^{T_c} e^{A_d T_c} d\tau\, B_d$

A_{dz}, B_{dz} and C_{dz} are a system/control matrix where Eq. (4-3) or (4-4) is discretized by the z operator each. These can be determined using the discretization method "zoh" or "matched" (matched pole-zero model [5]) using the MATLAB® function "c2dm". Note that the matched pole-zero model has the same poles as those of the step invariant model.

From Eqs. (4-5) and (4-6), the transfer function $G_d(\delta)$ of the discrete time model of disturbances can be expressed by the equations below:

$$G_d(\delta) = C_{d\delta}(\delta I_l - A_{d\delta})^{-1} B_{P\delta} = \frac{n_d(\delta)}{d_d(\delta)}$$

$$= \frac{n_{dl-1}\delta^{l-1} + \cdots n_{d1}\delta + n_{d0}}{\delta^{l} + d_{dl-1}\delta^{l-1} + \cdots d_{d1}\delta + d_{d0}} \quad (4\text{-}7)$$

Note that the numerator polynomial of the disturbance model $n_d(\delta)$ will not be used for compensator design.

4.1.3 Coprime Factorization of Compensator Design Model

To get ready for designing the compensator based on the parametrization of the stabilizing compensator, the compensator design model of Eq. (3-4) is expressed by the coprime factorization below. If the controlled object is a system with multiple inputs/outputs, the model needs to be expressed in right and left coprime factorizations. This controlled object is a system with single input/output, which means that the right coprime factorization is identical to the left coprime factorization. Therefore, this report simply uses the term "coprime factorization".

$$P(\delta) = N_P(\delta)/D_P(\delta) \quad (5\text{-}1)$$

$$N_P(\delta) = \frac{n_P(\delta)}{f(\delta)} = \frac{n_{Pn-1}\delta^{n-1} + \cdots + n_{P1}\delta + n_{P0}}{\delta^n + f_{n-1}\delta^{n-1} + \cdots + f_1\delta + f_0} \quad (5\text{-}2)$$

$$D_P(\delta) = \frac{d_P(\delta)}{f(\delta)} = \frac{\delta^n + d_{Pn-1}\delta^{n-1} + \cdots + d_{P1}\delta + d_{P0}}{\delta^n + f_{n-1}\delta^{n-1} + \cdots + f_1\delta + f_0} \quad (5\text{-}3)$$

$f(\delta)$ is a stable polynomial and any given parameter that can be set by the designer. When $f(\delta)$ is said to be stable, this means that its root exists at the center of the complex plane $-1/T_c$ within the circle of radius $1/T_c$ (see Fig. 9). How to set $f(\delta)$ will be described later.

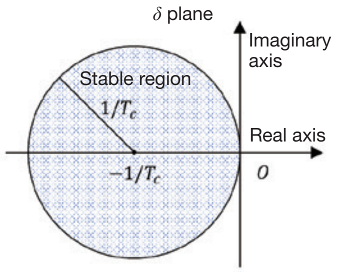


Fig. 9 Stable region of δ operator

4.1.4 Deriving a Compensator

For steering assist control, assume that the compensator $C_r(\delta)$ in Eqs. (1-1) and (1-2) is $C_r(\delta) = 0$. In this case, the control system can be represented by Fig. 10:

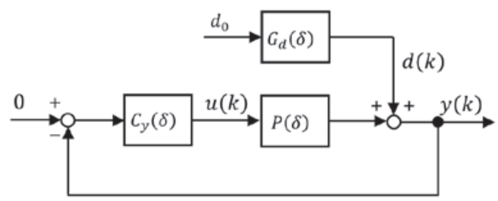


Fig. 10 Block diagram of steering assist control system

The compensator $C_y(\delta)$ based on the parametrization of the stabilizing compensator can be expressed by the equations below:

$$C_y(\delta) = N_C(\delta)/D_C(\delta) \quad (6\text{-}1)$$

$$N_C(\delta) = X_P(\delta) + R(\delta)D_P(\delta) \quad (6\text{-}2)$$

$$D_C(\delta) = Y_P(\delta) - R(\delta)N_P(\delta) \quad (6\text{-}3)$$

$$X_P(\delta) = \frac{n_X(\delta)}{g(\delta)} = \frac{n_{Xn-1}\delta^{n-1} + \cdots + n_{X1}\delta + n_{X0}}{\delta^{n-1} + g_{n-2}\delta^{n-2} + \cdots + g_1\delta + g_0} \quad (6\text{-}4)$$

$$Y_P(\delta) = \frac{n_Y(\delta)}{g(\delta)} = \frac{n_{Yn-1}\delta^{n-1} + \cdots + n_{Y1}\delta + n_{Y0}}{\delta^{n-1} + g_{n-2}\delta^{n-2} + \cdots + g_1\delta + g_0} \quad (6\text{-}5)$$

$g(\delta)$ is a stable polynomial that is any given parameter to be set by the designer. $R(\delta)$ is a stable-proper transfer function (degree of numerator polynomial is equal to or less than degree of denominator polynomial) that is a free parameter to be selected by the designer. How to set these parameters will be described later.

$X_p(\delta)$ and $Y_p(\delta)$ are solutions to Bezout equations.

$$X_P(\delta)N_P(\delta) + Y_P(\delta)D_P(\delta) = 1 \quad (6\text{-}6)$$

Firstly, $X_p(\delta)$ and $Y_p(\delta)$ should be derived. Multiplying both sides of Eq. (6-6) by $h(\delta) = f(\delta)g(\delta)$ yields an identical equation below:

$$n_X(\delta)n_P(\delta) + n_Y(\delta)d_p(\delta) = h(\delta) \quad (6\text{-}7)$$

where

$h(\delta) = \delta^{2n-1} + h_{2n-2}\delta^{2n-2} + \ldots + h_1\delta + h_0$

When $n_p(\delta)$ and $d_p(\delta)$ do not share a common divisor, $n_x(\delta)$ and $n_y(\delta)$ that satisfy Eq. (6-7) will be uniquely decided. Their coefficient can be determined using the equation below [5]:

$$\Theta^T = \Psi^T E^{-1} \quad (6\text{-}8)$$

where

$$\Theta^T = [n_{Y0} \ \cdots \ n_{Yn-1} \ n_{X0} \ \cdots \ n_{Xn-1}]$$
$$\Psi^T = [h_0 \ h_1 \ \cdots \ h_{2n-2} \ 1]$$
$$E = \begin{bmatrix} d_{P0} & d_{P1} & \cdots & d_{Pn-1} & 1 & 0 & \cdots & 0 & 0 \\ 0 & d_{P0} & \cdots & d_{Pn-2} & d_{Pn-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & d_{P0} & d_{P1} & d_{P2} & \cdots & d_{Pn-1} & 1 \\ n_{P0} & n_{P1} & \cdots & n_{Pn-1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & n_{P0} & \cdots & n_{Pn-2} & n_{Pn-1} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & n_{P0} & n_{P1} & n_{P2} & \cdots & n_{Pn-1} & 0 \end{bmatrix}$$

Next is the process to derive $R(\delta)$. In Fig. 10, the control output $y(k)$ in response to the disturbance $d(k)$ can be expressed by the equation below:

$$y(k) = S(\delta)d(k) = D_C(\delta)D_P(\delta)G_d(\delta)d_0 \tag{7-1}$$

$G_d(\delta)$ may have unstable poles or a pole that can delay the attenuation of $d(k)$ regardless of its stability. In order to minimize fluctuations of $y(k)$ due to the effect of such poles as quickly as possible, the zero of $D_c(\delta)$ (the root of the numerator polynomial) must include the poles of $G_d(\delta)$. Therefore, it is necessary to select $R(\delta)$ so that the numerator polynomial of $D_c(\delta)$ includes the denominator polynomial $d_d(\delta)$ of $G_d(\delta)$. Since the degree of the disturbance model of Eq. (4-7) is l, the degree of $R(\delta)$ is set to l-1. $R(\delta)$ is expressed by the equation below:

$$R(\delta) = \frac{n_R(\delta)}{d_R(\delta)} = \frac{n_{Rl-1}\delta^{l-1} + n_{Rl-2}\delta^{l-2} + \cdots + n_{R1}\delta + n_{R0}}{\delta^{l-1} + d_{Rl-2}\delta^{l-2} + \cdots + d_{R1}\delta + d_{R0}} \tag{7-2}$$

where $d_R(\delta)$ is a stable polynomial. How to set the polynomial will be described later.

Developing $D_c(\delta)$ yields the equation below:

$$D_C(\delta) = Y_P(\delta) - R(\delta)N_P(\delta) = \frac{n_Y(\delta)}{g(\delta)} - \frac{n_R(\delta)}{d_R(\delta)}\frac{n_P(\delta)}{f(\delta)}$$
$$= \frac{d_R(\delta)f(\delta)n_Y(\delta) - g(\delta)n_P(\delta)n_R(\delta)}{d_R(\delta)f(\delta)g(\delta)} \tag{7-3}$$

The following identical equation should be set so that the numerator polynomial of Eq. (7-3) includes $d_d(\delta)$:

$$d_R(\delta)f(\delta)n_Y(\delta) - g(\delta)n_P(\delta)n_R(\delta) = d_d(\delta)q(\delta) \tag{7-4}$$

Modifying Eq. (7-4) yields the equation below:

$$d_d(\delta)q(\delta) + w(\delta)n_R(\delta) = \gamma(\delta) \tag{7-5}$$

where

$$w(\delta) = g(\delta)n_P(\delta) = \delta^{2n-2} + w_{2n-3}\delta^{2n-3} + \ldots + w_1\delta + w_0$$
$$\gamma(\delta) = d_R(\delta)f(\delta)n_Y(\delta) = \delta^{2n+l-2} + \gamma_{2n+l-3}\delta^{2n+l-3} + \ldots + \gamma_1\delta + \gamma_0$$
$$q(\delta) = q_{2n-2}\delta^{2n-2} + q_{2n-3}\delta^{2n-3} + \ldots + q_1\delta + q_0$$

The coefficient of $n_R(\delta)$ can be determined using the equation below:

$$\Theta^T = \Psi^T E^{-1} \tag{7-6}$$

where

$$\Theta^T = [q_0 \ q_1 \ \cdots \ q_{2n-2} \ n_{R0} \ n_{R1} \ \cdots \ n_{Rl-1}]$$
$$\Psi^T = [\gamma_0 \ \gamma_1 \ \cdots \ \gamma_{2n+l-3} \ 1]$$
$$E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$
$$E_1 = \begin{bmatrix} d_{d0} & d_{d1} & \cdots & d_{dl-1} & 1 & 0 & \cdots & 0 & 0 \\ 0 & d_{d0} & \cdots & d_{dl-2} & d_{dl-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & d_{d0} & d_{d1} & d_{d2} & \cdots & d_{dl-1} & 1 \end{bmatrix}$$

: $(2n-1)\times(2n+l-1)$ matrix

$$E_2 = \begin{bmatrix} w_0 & w_1 & \cdots & w_{2n-2} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & w_0 & \cdots & w_{2n-3} & w_{2n-2} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & w_0 & w_2 & \cdots & w_{2n-2} & \cdots & 0 \end{bmatrix}$$

: $l\times(2n+l-1)$ matrix

4.1.5 Example of Basic Design of Compensator

(1) Determining the compensator design model

When A_p, B_p, C_p, E_p, and F_p in Eqs. (2-3) and (2-4) are given their physical parameter values and then discretized, the transfer function of Eq. (3-3) will be:

$$P(\bar{\delta}) = \frac{n_P(\bar{\delta})}{d_P(\bar{\delta})} = \frac{n_{P1}\bar{\delta} + n_{P0}}{\bar{\delta}^2 + d_{P1}\bar{\delta} + d_{P0}}$$
$$= \frac{7.807\times10^{-3}(\bar{\delta}+1.980)}{\bar{\delta}^2 + 7.964\times10^{-2}\bar{\delta} + 2.163\times10^{-2}} \tag{8-1}$$

The δ operator and the coefficients of the numerator and denominator of Eq. (8-1) were made dimensionless with the inverse of the control period $1/T_c$ (hereinafter "the control frequency"). In other words, $\bar{\delta}$ in the equation is $\bar{\delta} = T_c\delta$. All figures appearing in the following examples indicate dimensionless transfer functions.

The poles and zeros of Eq. (8-1) are all within the stable region, implying a stable controlled object. Note that the zeros in Eq. (8-1) have been added through discretization.

Next, the coprime factorization of Eq. (8-1) $P(\bar{\delta}) = N_p(\bar{\delta}) / D_p(\bar{\delta})$ is set as follows:

$$N_P(\bar{\delta}) = \frac{n_P(\bar{\delta})}{f(\bar{\delta})} = \frac{7.807\times10^{-3}(\bar{\delta}+1.980)}{(\bar{\delta}+0.2583)^2} \tag{8-2}$$

$$D_P(\bar{\delta}) = \frac{d_P(\bar{\delta})}{f(\bar{\delta})} = \frac{\bar{\delta}^2 + 7.964\times10^{-2}\bar{\delta} + 2.163\times10^{-2}}{(\bar{\delta}+0.2583)^2} \tag{8-3}$$

$1/f(\bar{\delta})$ in Eqs. (8-2) and (8-3) is the matched pole-zero model of the following stable transfer function $1/f(s)$ that has been made dimensionless with the control frequency $1/T_c$:

$$1/f(s) = 1/(s^2 + 2\zeta_f\omega_f s + \omega_f^2) \tag{8-4}$$

where $0 < \omega_f$, $0 < \zeta_f$

If $1/f(s)$ is stable, its matched pole-zero model $1/f(\bar{\delta})$is also stable. To allow the control system to deliver high responsivity, it is desirable to set ω_f to a value as high as possible. ζ_f, which is the attenuation factor of the poles of the control system, should be set to 1 or more. For convenience, these are set as follows:

$\zeta_f = 1$, $\omega_f = 2\sqrt{K_{pe}/I_{pe}}$ (natural angular frequency decided by the equivalent stiffness and equivalent inertia moment of the pinion shaft multiplied by 2).

The solutions to Bezout equations $X_p(\delta)$ and $Y_p(\delta)$ can be expressed in the equations below:

$$X_P(\bar{\delta}) = \frac{n_X(\bar{\delta})}{g(\bar{\delta})} = \frac{8.179\bar{\delta} + 0.2314}{\bar{\delta} + 0.2583} \quad (8\text{-}5)$$

$$Y_P(\bar{\delta}) = \frac{n_Y(\bar{\delta})}{g(\bar{\delta})} = \frac{\bar{\delta} + 0.6314}{\bar{\delta} + 0.2583} \quad (8\text{-}6)$$

$1/g(\delta)$ in Eqs. (8-5) and (8-6) is the matched pole-zero model of the following stable transfer function $1/g(s)$ that has been made dimensionless with the control frequency $1/T_c$:

$$1/g(s) = 1/(s + \omega_g) \quad (8\text{-}7)$$

where $\omega_g = 2\sqrt{K_{TB}/I_{pe}}$ is used.

(2) Setting disturbance models

The disturbance model for the continuous time system is set as follows. The numerator polynomial $n_d(s)$ of the disturbance model is hereinafter omitted because it will not be used for compensator design.

$$G_d(s) = \frac{n_d(s)}{d_d(s)} = \frac{n_d(s)}{s^2 + 2\zeta_d\omega_d s + \omega_d^{\,2}} \quad (9\text{-}1)$$

where $\zeta_d = 1$. The following settings will be used according to the steering assist torque level:

$\omega_d = 0.5\sqrt{K_{pe}/I_{pe}}$: Large steering assist torque
$\omega_d = 0.9\sqrt{K_{pe}/I_{pe}}$: Medium steering assist torque
$\omega_d = 1.5\sqrt{K_{pe}/I_{pe}}$: Small steering assist torque

In these settings, the step invariant or matched pole-zero model of the transfer function in Eq. (9-1) that has been made dimensionless with the control frequency $1/T_c$ can be expressed by the equation below:

$$G_d(\bar{\delta}) = \frac{n_d(\bar{\delta})}{d_d(\bar{\delta})} = \frac{n_d(\bar{\delta})}{\bar{\delta}^2 + 2\zeta_d\bar{\omega}_d\bar{\delta} + \omega_d^{\,2}} = \frac{n_d(\bar{\delta})}{(\bar{\delta} + \bar{\omega}_d)^2} \quad (9\text{-}2)$$

where $\zeta_d = 1$. $\bar{\omega}_d$ is as follows:

$\bar{\omega}_d = 0.07198$: Large steering assist torque
$\bar{\omega}_d = 0.1258$: Medium steering assist torque
$\bar{\omega}_d = 0.2008$: Small steering assist torque

(3) Deriving free parameter $R(\bar{\delta})$

It is assumed that the pole $d_R(\bar{\delta})$ of the free parameter is the same as $g(\bar{\delta})$ of Eqs. (8-5) and (8-6). Substituting Eqs. (8-2), (8-6), and (9-2) as well as $d_R(\bar{\delta}) = g(\bar{\delta}) = \bar{\delta} + 0.2583$ in Eq. (7-6) yields the following free parameter $R(\delta)$:

$$R(\bar{\delta}) = \frac{n_R(\bar{\delta})}{d_R(\bar{\delta})} = \frac{\bar{n}_{R1}\bar{\delta} + \bar{n}_{R0}}{\bar{\delta} + 0.2583} \quad (9\text{-}3)$$

where

$\bar{n}_{R1} = 15.640$, $\bar{n}_{R0} = 2.429$: Large steering assist torque
$\bar{n}_{R1} = 10.135$, $\bar{n}_{R0} = 1.888$: Medium steering assist torque
$\bar{n}_{R1} = 3.747$, $\bar{n}_{R0} = 0.8549$: Small steering assist torque

(4) Frequency characteristics of control system

From the above, substituting Eqs. (8-2), (8-3), (8-5), (8-6), and (9-3) in Eqs. (6-1) to (6-3) yields $C_y(\delta)$ as follows:

$$C_y(\bar{\delta}) = \frac{23.819(\bar{\delta} + 0.121)(\bar{\delta}^2 + 0.2204\bar{\delta} + 2.358 \times 10^{-2})}{(\bar{\delta} + 0.8819)(\bar{\delta} + 0.07198)^2}$$

: Large steering assist torque (10-1)

$$C_y(\bar{\delta}) = \frac{18.314(\bar{\delta} + 0.1513)(\bar{\delta}^2 + 0.2392\bar{\delta} + 2.031 \times 10^{-2})}{(\bar{\delta} + 0.8172)(\bar{\delta} + 0.1258)^2}$$

: Medium steering assist torque (10-2)

$$C_y(\bar{\delta}) = \frac{11.926(\bar{\delta} + 0.205)(\bar{\delta} + 0.1938)(\bar{\delta} + 0.07162)}{(\bar{\delta} + 0.7172)(\bar{\delta} + 0.2008)^2}$$

: Small steering assist torque (10-3)

According to the equations above, the poles of $C_y(\delta)$ include the pole $d_d(\bar{\delta})$ of the disturbance model in Eq. (9-2) for all the torque cases. Fig. 11 shows the poles and zeros of the compensator of Eqs. (10-1) to (10-3) plotted on a complex plane. The figure indicates that all the poles and zeros are within the stable region. Note that both the vertical and horizontal axes have been made dimensionless with the control frequency $1/T_c$.

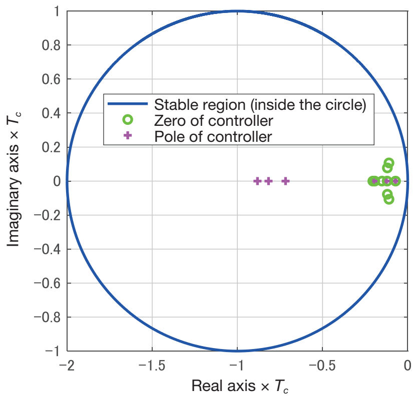
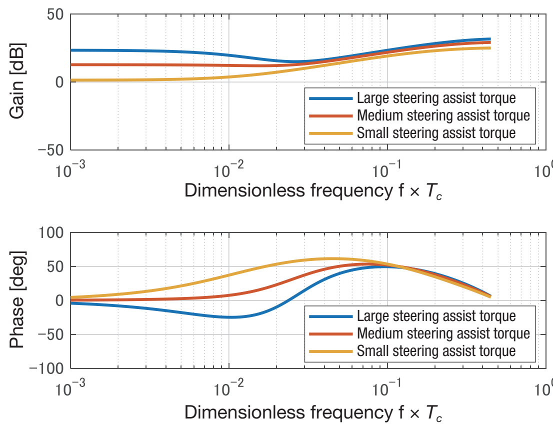


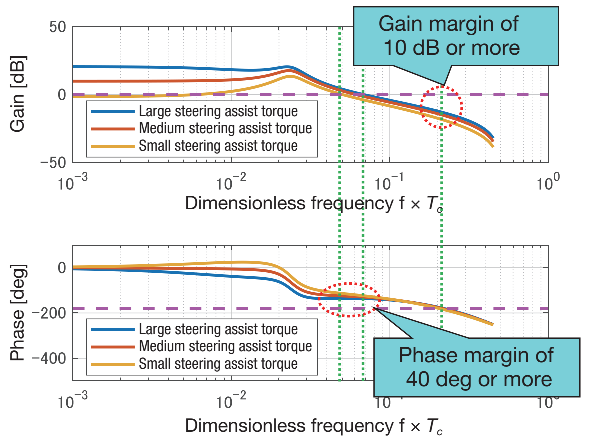
Fig. 11 Poles and zeros of compensator (basic design)

Next, the frequency characteristics of $C_y(\bar{\delta})$, $C_y(\bar{\delta})P(\bar{\delta})$ (loop transfer function), and $S(\bar{\delta})$ are shown in Fig. 12. For plotting of the Bode diagrams in Fig. 12, $C_y(\bar{\delta})$, $C_y(\bar{\delta})P(\bar{\delta})$, and $S(\bar{\delta})$ have been converted into the form of the z operator and applied with the MATLAB® function "dbode". The horizontal axis of these diagrams has been made dimensionless with the control frequency $1/T_c$. According to the diagrams, the gain of the compensator or loop transfer function depends on the steering assist torque level.

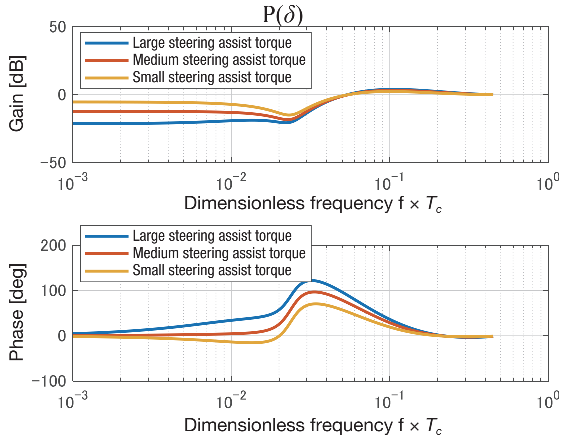
This design approach is not intended to design compensators with a focus on the loop transfer function or stability margin. Still, it should be noted that eventually the



(a) Frequency characteristics of compensator Cy(δ)



(b) Frequency characteristics of loop transfer function Cy(δ) P(δ)



(c) Frequency characteristics of transfer function S(δ)

Fig. 12 Frequency characteristics of steering assist control system

gain margin is not less than 10 dB and the phase margin not less than 40 degrees for all cases.

$S(\bar{\delta})$, which is called the sensitivity function, is less likely to be affected by the parameter variations of the controlled object or disturbances with its smaller gain. The smaller gain means a smaller control output to the steering wheel rotation, in other words, a larger steering assist torque. The poles of $S(\bar{\delta})$ are the poles $d_{cl}(\bar{\delta})$ of the closed-loop transfer function of the control system in Fig. 10 and can be expressed by the equation below:

$$d_{cl}(\bar{\delta}) = \{f(\bar{\delta})\}^2 g(\bar{\delta}) d_R(\bar{\delta}) = (\bar{\delta} + 0.2583)^6 \qquad (10\text{-}4)$$

That is, $f(\bar{\delta})$, $g(\bar{\delta})$ and $d_R(\bar{\delta})$ that were set during design become the poles of the closed-loop transfer function. This implies that this design approach is to design a compensator by setting the poles of the closed-loop transfer function (or poles of the control system).

4.2 Basic Design of Steering Angle Control Compensator

4.2.1 Deriving a Compensator Design Model

Fig. 13 shows a model of EPS including the steering wheel. For steering angle control, the steering angle determined from the motor rotation angle must follow the target steering angle signal while suppressing vibration caused by the turn of the steering wheel. Therefore, the controlled object is the steering system including the rotary motion of the equivalent inertia moment of the steering wheel shaft. The figure uses symbols to indicate the following meaning. The other symbols not on the list below have the same meaning as those used in Fig. 8.

θ_m : Motor rotation angle [rad] ($\theta_m = R_P\theta_P$)

I_h : Equivalent inertia moment of steering wheel shaft [kg·m²]

C_h : Equivalent viscous resistance coefficient of steering wheel shaft [N·m/(rad/s)]

τ_h : Steering wheel input torque [N·m] (assume $\tau_h = 0$ with the driver's hands off the wheel)

The control output, namely, the steering angle signal $\hat{\theta}_p$ can be determined using the equation below:

$$\hat{\theta}_p = \theta_m / R_w \qquad (11\text{-}1)$$

Assuming again that i_m is equal to u (the actual current completely follows the motor current command) and that all the parts except the torsion bar are rigid bodies, the rotary motion of the pinion can be expressed by the differential equation below:

$$I_{pe}\ddot{\theta}_p + C_{pe}\dot{\theta}_p = R_w K_t i_m + K_{TB}(\theta_h - \theta_p) - K_r R_p^2 \theta_p \qquad (11\text{-}2)$$

where

$i_m = u$

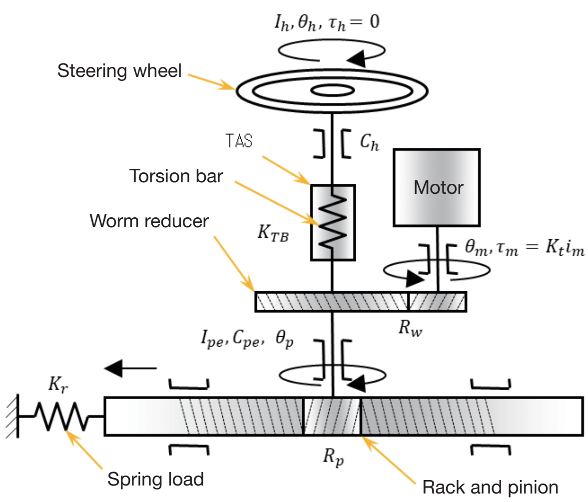


Fig. 13 Model of EPS including steering wheel

The rotary motion of the wheel can be expressed by the equation below:

$$I_h\ddot{\theta}_h + C_h\dot{\theta}_h = -K_{TB}(\theta_h - \theta_p) \quad (11\text{-}3)$$

From Eqs. (11-1) to (11-3), the following state and output equations can be obtained:

$$\dot{x} = A_P x + B_P u \quad (11\text{-}4)$$

$$y = C_P x \quad (11\text{-}5)$$

where

$x = [\theta_p \quad \theta_h \quad \dot{\theta}_p \quad \dot{\theta}_h]^T$: State variable

$y = \hat{\theta}_p = \theta_m / R_w = \theta_p$: Control output

$$A_P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_{pe}/I_{pe} & K_{TB}/I_{pe} & -C_{pe}/I_{pe} & 0 \\ K_{TB}/I_h & -K_{TB}/I_h & 0 & -C_h/I_h \end{bmatrix}$$

$$B_P = \begin{bmatrix} 0 \\ 0 \\ R_w K_t / I_{pe} \\ 0 \end{bmatrix}, \quad C_P = [1 \quad 0 \quad 0 \quad 0]$$

In Eqs. (11-4) and (11-5), the state and output equations for the discrete time model can be expressed by the equations below:

$$\delta x(k) = A_{P\delta} x(k) + B_{P\delta} u(k) \quad (11\text{-}6)$$

$$y(k) = C_{P\delta} x(k) \quad (11\text{-}7)$$

where

$A_{P\delta} = (A_{Pz} - I_4)/T_c$, I_4 : 4 × 4 unit matrix

$B_{P\delta} = B_{Pz}/T_c$, $C_{P\delta} = C_{Pz} = C_P$

$A_{Pz} = e^{A_P T_c}$, $B_{Pz} = \int_0^{T_c} e^{A_P T_c} d\tau \, B_P$

When Eqs. (11-6) and (11-7) are selected as the compensator design model, the transfer function $P(\delta)$ in Figs. 5 and 6 can be expressed by the equation below:

$$P(\delta) = C_{P\delta}(\delta I_4 - A_{P\delta})^{-1} B_{P\delta} = \frac{n_P(\delta)}{d_P(\delta)} = \frac{n_{P3}\delta^3 + n_{P2}\delta^2 n_{P1}\delta + n_{P0}}{\delta^4 + d_{P3}\delta^3 + d_{P2}\delta^2 + d_{P1}\delta + d_{P0}} \quad (11\text{-}8)$$

The setting of disturbance models and coprime factorization of the compensator design model are omitted here as these are as described in sections 4.1.2 and 4.1.3. The following describes how to derive a compensator.

4.2.2 Setting a Target Value Signal Model

This section discusses the target value signal model of degree l represented by the equation below:

$$G_r(\delta) = \frac{n_r(\delta)}{d_r(\delta)} = \frac{n_{dl-1}\delta^{l-1} + \cdots n_{d1}\delta + n_{d0}}{\delta^l + d_{dl-1}\delta^{l-1} + \cdots d_{d1}\delta + d_{d0}} \quad (12)$$

Note that the numerator polynomial of the target value signal model $n_r(\delta)$ will not be used for compensator design.

4.2.3 Deriving a Compensator

For steering angle control, assume that the compensator $C_r(\delta)$ of Eqs. (1-1) and (1-2) is $C_r(\delta) = F_c(\delta) / D_c(\delta)$ based on the parametrization [3] of the stabilizing compensator for the two-degree-of-freedom control system. $F_c(\delta)$ is a stable-proper transfer function (hereinafter a "pre-compensator"). In this case, the control system can be represented by Fig. 14:

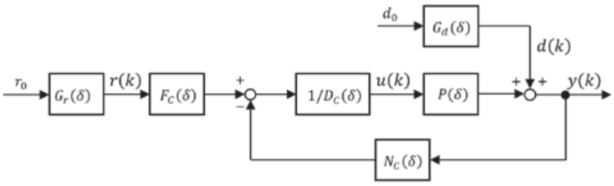


Fig. 14 Block diagram of steering angle control system

In the control system shown in Fig. 14, the transfer function $G_{ry}(\delta)$ ranging from the target steering angle signal $r(k)$ to the control output y (hereinafter "target tracking characteristics") can be expressed by the equation below:

$$G_{ry}(\delta) = F_C(\delta) N_P(\delta) \quad (13\text{-}1)$$

Then, $G_{ry}(\delta)$ is set as follows:

$$G_{ry}(\delta) = \frac{n_M(\delta)}{d_M(\delta)} = \frac{n_{Mm-1}\delta^{m-1} + n_{Mm-2}\delta^{m-2} + \cdots + n_{M1}\delta + n_{M0}}{\delta^m + d_{Mm-1}\delta^{m-1} + \cdots + d_{M1}\delta + d_{M0}} \quad (13\text{-}2)$$

where $d_M(\delta)$ is a stable polynomial.

Next, the deviation $e_r(k)$ of the output $y_r(k)$ of $G_{ry}(\delta)$ from $r(k)$ can be expressed by the equation below:

$$\begin{aligned} e_r(k) &= r(k) - y_r(k) = r(k) - G_{ry}(\delta) r(k) \\ &= \{1 - G_{ry}(\delta)\} r(k) = \frac{d_M(\delta) - n_M(\delta)}{d_M(\delta)} r(k) \\ &= \frac{d_M(\delta) - n_M(\delta)}{d_M(\delta)} G_r(\delta) r_0 \end{aligned} \quad (13\text{-}3)$$

$d_M(\delta)$'s degree m may be any number, but needs to be set from the following perspectives:

① If the compensator design model has an unstable zero, in other words, if $n_p(\delta)$ or $d_r(\delta)$ has an unstable root, $n_M(\delta)$ must include the root.

② In order for the deviation $e_r(k)$ to converge to zero (0) asymptotically, $\{d_M(\delta) - n_M(\delta)\}$ in Eq. (13-3) must have the unstable pole of $G_r(\delta)$ as its root.

Then, assuming that $n_p(\delta)$ has an unstable root, an identical equation shown below is set so that $\{d_M(\delta) - n_M(\delta)\}$ in Eq. (13-3) has the pole $d_r(\delta)$ of Eq. (12) as a divisor:

$$d_M(\delta) - n_P(\delta)\beta_M(\delta) = d_r(\delta)\alpha_M(\delta) \quad (13\text{-}4)$$

where

$$n_M(\delta) = n_P(\delta)\beta_M(\delta) \quad (13\text{-}5)$$

$\alpha_M(\delta)$ and $\beta_M(\delta)$ are polynomials of degree n-1 and l-1, respectively. Degree of $d_M(\delta)$ is $m = n + l - 1$. Modifying Eq. (13-4) gives the equation below:

$$n_P(\delta)\beta_M(\delta) + d_r(\delta)\alpha_M(\delta) = d_M(\delta) \quad (13\text{-}6)$$

The coefficient of $\beta_M(\delta)$ can be determined using the equation below:

$$\Theta^T = \Psi^T E^{-1} \quad (13\text{-}7)$$

where

$$\Theta^T = [\alpha_{M0} \ \cdots \ \alpha_{Mn-1} \ \beta_{M0} \ \cdots \ \beta_{Ml-1}]$$

$$\Psi^T = [d_{M0} \ d_{M1} \ \cdots \ d_{Mn+l-2} \ 1]$$

$$E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} d_{r0} & d_{r1} & \cdots & d_{rl-1} & 1 & 0 & \cdots & 0 & 0 \\ 0 & d_{r0} & \cdots & d_{rl-2} & d_{rl-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & d_{r0} & d_{r1} & d_{r2} & \cdots & d_{rl-1} & 1 \end{bmatrix}$$

$: n \times (n+l)$ matrix

$$E_2 = \begin{bmatrix} n_{P0} & n_{P1} & \cdots & n_{Pn-1} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & n_{P0} & \cdots & n_{Pn-2} & n_{Pn-1} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & n_{P0} & n_{P2} & \cdots & n_{Pn-1} & \cdots & 0 \end{bmatrix}$$

$: l \times (n+l)$ matrix

How to set $d_M(\delta)$ will be described later.

Substituting Eq. (13-2) in Eq. (13-1) and modifying it gives the equation below:

$$F_C(\delta) = \frac{f(\delta)n_M(\delta)}{n_P(\delta)d_M(\delta)} \tag{13-8}$$

Substituting the relation of Eq. (13-5) in Eq. (13-8) yields the equation below:

$$F_C(\delta) = \frac{f(\delta)\beta_M(\delta)}{d_M(\delta)} \tag{13-9}$$

Eq. (13-9) can be used to determine a pre-compensator $F_c(\delta)$.

A description of how to derive the compensator $C_y(\delta)$ is omitted here as it is the same as for steering assist control. The next section provides an example of basic design of the compensator.

4.2.4 Example of Basic Design of Compensator

(1) Determining a compensator design model

When A_p, B_p, C_p, E_p, and F_p in Eqs. (11-4) and (11-5) are input with their physical parameter values, discretized and made dimensionless with the control frequency $1/T_c$, the transfer function of Eq. (11-8) is as follows:

$$P(\bar{\delta}) = \frac{n_P(\bar{\delta})}{d_P(\bar{\delta})} \tag{14-1}$$

where

$$n_P(\bar{\delta}) = 3.406\times10^{-5}(\bar{\delta} + 1.980) \times(\bar{\delta}^2 + 8.741\times10^{-3}\bar{\delta} + 5.718\times10^{-5})$$

$$d_P(\bar{\delta}) = \bar{\delta}(\bar{\delta} + 1.233\times10^{-2}) \times(\bar{\delta}^2 + 7.599\times10^{-2}\bar{\delta} + 2.685\times10^{-2})$$

Fig. 15 shows the poles and zeros of $P(\delta)$ in Eq. (14-1) plotted on a complex plane. With a focus on the numerator of Eq. (14-1), there are three zeros. Of these, a real root is a zero that has been added through discretization. The remaining complex roots correspond to the original continuous time system. These complex roots are what make the resonance point (generally with a low attenuation factor), which is decided by the equivalent inertia moment of the steering wheel shaft and the stiffness of the torsion bar, appear as an antiresonance point in the zeros of $P(\delta)$. Causing either compensator $C_r(\delta)$ or $C_y(\delta)$ to have these zeros as divisors in its poles could prevent the antiresonance point from appearing in the zeros of the transfer characteristics ranging from the target steering angle signal $r(k)$ to the control output $y(k)$. In this case, however, the antiresonance point inevitably appears as a pole of the transfer characteristics from $r(k)$ to the wheel rotation angle to be a resonance point. So, if the target steering angle signal abruptly changes, vibration may occur as the steering wheel is turned. Then, the compensator should be so designed that the poles of the compensator cannot have the zeros of the compensator design model as their divisors. This can be achieved by determining the compensator $F_c(\delta)$ using Eq. (13-9) and making settings so that the denominator polynomial $f(\delta)$ in Eqs. (5-2) and (5-3), the denominator polynomial $g(\delta)$ in Eqs. (6-4) and (6-5), and the denominator polynomial $d_R(\delta)$ in Eq. (7-2) cannot have the zeros of the compensator design model as their divisors.

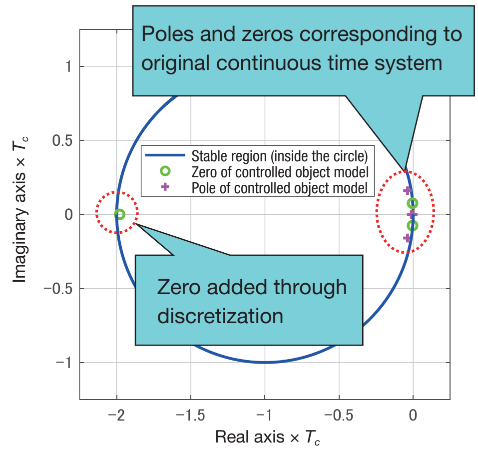


Fig. 15 Poles and zeros of compensator design model

Next is to set the coprime factorization $P(\bar{\delta}) = N_p(\bar{\delta}) / D_p(\bar{\delta})$ in Eq. (14-1) as follows:

$$N_P(\bar{\delta}) = \frac{n_P(\bar{\delta})}{f(\bar{\delta})} \tag{14-2}$$

$$D_P(\bar{\delta}) = \frac{d_P(\bar{\delta})}{f(\bar{\delta})} \tag{14-3}$$

$1/f(\bar{\delta})$ in Eqs. (14-2) and (14-3) are the matched pole-zero model of the following stable transfer function $1/f(s)$ that has been made dimensionless with the control frequency $1/T_c$:

$$1/f(s) = 1/\ \{f_1(s)f_2(s)\} \tag{14-4}$$

where

$$f_1(s) = s^2 + 2\zeta_{f1}\omega_{f1}s + \omega_{f1}^2$$

$$f_2(s) = s^2 + 2\zeta_{f2}\omega_{f1}s + \omega_{f2}^2$$

$$0<\omega_{f1},\ \ 0<\omega_{f2},\ \ 0<\zeta_{f1},\ \ 0<\zeta_{f2}$$

where $\zeta_{f1} = \zeta_{f2} = 2.3$ and $\omega_{f1} = \omega_{f2} = 0.7\sqrt{K_{pe}/I_{pe}}$. Reducing ζ_{f1} and ζ_{f2} or raising ω_{f1} and ω_{f2} will tend to cause the controller to have unstable poles. So, these parameters are set to smaller values than for steering assist control.

The solutions to Bezout equations $X_p(\bar{\delta})$ and $Y_p(\bar{\delta})$ can be expressed by the equations below:

$$X_P(\bar{\delta}) = \frac{n_X(\bar{\delta})}{g(\bar{\delta})} \qquad (14\text{-}5)$$

$$Y_P(\bar{\delta}) = \frac{n_Y(\bar{\delta})}{g(\bar{\delta})} \qquad (14\text{-}6)$$

where

$$n_X(\bar{\delta}) = 1910.8(\bar{\delta} + 0.4936) \times (\bar{\delta}^2 + 2.550\times10^{-2}\bar{\delta} + 1.781\times10^{-4})$$

$$n_Y(\bar{\delta}) = (\bar{\delta} + 0.5236)(\bar{\delta} + 0.5953) \times (\bar{\delta} + 1.284\times10^{-3})$$

$$g(\bar{\delta}) = (\bar{\delta} + 2.364\times10^{-2})(\bar{\delta} + 9.930\times10^{-2}) \times (\bar{\delta} + 0.3669)$$

$1/g(\bar{\delta})$ is the matched pole-zero model of the following stable transfer function $1/g(s)$ that has been made dimensionless with the control frequency $1/T_c$:

$$1/g(s) = 1/\ \{g_1(s)g_2(s)\} \qquad (14\text{-}7)$$

where

$$g_1(s) = s + \omega_{g1}$$

$$g_2(s) = s^2 + 2\zeta_{g2}\omega_{g1}s + \omega_{g2}^2$$

where $\zeta_{g2} = 2.3$, $\omega_{g1} = \omega_{g2} = 0.7\sqrt{K_{TB}/I_{pe}}$.

(2) Setting target value signal models

To ensure that the control output tracking to the target steering angle signal, which changes in ramp rate, converges to zero (0) asymptotically, a target value signal model is set as follows:

$$G_r(\bar{\delta}) = \frac{n_r(\bar{\delta})}{d_r(\bar{\delta})} = \frac{1}{\bar{\delta}^2} \qquad (15)$$

(3) Setting disturbance models

To ensure that the control output tracking disturbances, which change in ramp rate, converges to zero (0) asymptotically, a disturbance model for the continuous time system is set as follows:

$$G_d(s) = \frac{n_d(s)}{d_d(s)} = \frac{1}{s^2} \qquad (16\text{-}1)$$

A step invariant model or matched pole-zero model of the transfer function of Eq. (16-1) that has been made into a dimensionless transfer function with the control frequency $1/T_c$ can be expressed by the equation below:

$$G_d(\bar{\delta}) = \frac{n_d(\bar{\delta})}{d_d(\bar{\delta})} = \frac{1}{\bar{\delta}^2} \qquad (16\text{-}2)$$

(4) Deriving free parameter $R(\bar{\delta})$

It is assumed that the pole $d_R(\bar{\delta})$ of the free parameter is $d_R(\bar{\delta}) = \bar{\delta} + 9.930 \times 10\text{-}2$, which is the same as one of the divisors of $g(\bar{\delta})$ in Eqs. (14-5) and (14-6). Substituting Eqs. (14-2), (14-6) and (15-2) as well as $d_R(\bar{\delta})$ in Eq. (7-6) gives the following free parameter $R(\bar{\delta})$:

$$R(\bar{\delta}) = \frac{n_R(\bar{\delta})}{d_R(\bar{\delta})} = \frac{6590\bar{\delta} - 9.002}{\bar{\delta} + 9.930\times10^{-2}} \qquad (16\text{-}3)$$

(5) Deriving compensator $F_c(\bar{\delta})$

The target tracking characteristics are set as follows:

$$G_{ry}(\bar{\delta}) = \frac{n_M(\bar{\delta})}{d_M(\bar{\delta})} = \frac{n_P(\bar{\delta})\beta_M(\bar{\delta})}{d_M(\bar{\delta})} \qquad (16\text{-}4)$$

where

$$n_M(\bar{\delta}) = 1.601\times10^{-3}(\bar{\delta} + 8.925\times10^{-3}) \times (\bar{\delta} + 1.980)(\bar{\delta}^2 + 8.741\bar{\delta} + 5718)$$

$$d_M(\bar{\delta}) = (\bar{\delta} + 4.383\times10^{-2})^5$$

$1/d_M(\delta)$ in Eq. (16-4) is the matched pole-zero model of the following stable transfer function $1/d_M(s)$ that has been made dimensionless with the control frequency $1/Tc$:

$$1/d_M(s) = 1/\ \{d_{M1}(s)d_{M2}(s)d_{M3}(s)\} \qquad (16\text{-}5)$$

where

$$d_{M1}(s) = s + \omega_{M1}$$

$$d_{M2}(s) = s^2 + 2\zeta_{M2}\omega_{M1}s + \omega_{M2}^2$$

$$d_{M3}(s) = s^2 + 2\zeta_{M3}\omega_{M3}s + \omega_{M3}^2$$

$$0<\omega_{M1},\ 0<\omega_{M2},\ 0<\omega_{M3},\ 0<\zeta_{M2},\ 0<\zeta_{M3}$$

where $\zeta_{M2} = \zeta_{M3} = 1$ and $\omega_{M1} = \omega_{M2} = \omega_{M3} = 0.3 \text{ x} \sqrt{K_{pe}/I_{pe}}$. $\beta_M(\bar{\delta})$ in Eq. (16-4) has been determined by substituting $n_p(\bar{\delta})$ of Eq. (14-1), $d_r(\bar{\delta})$ of Eq. (15), and $d_M(\bar{\delta})$ of Eq. (17-2) in Eq. (13-7).

When $f(\bar{\delta})$ of Eqs. (14-2) and (14-3) and $\beta_M(\bar{\delta})$ and $d_M(\bar{\delta})$ of Eq. (16-4) are substituted in Eq. (13-9), the compensator $F_c(\bar{\delta})$ can be expressed by the equation below:

$$F_C(\bar{\delta}) = \frac{n_F(\bar{\delta})}{d_F(\bar{\delta})} \qquad (16\text{-}6)$$

where

$$n_F(\bar{\delta}) = 46.993(\bar{\delta} + 8.925\times10^{-3}) \times (\bar{\delta} + 2.364\times10^{-2})^2(\bar{\delta} + 0.3669)^2$$

$$d_F(\bar{\delta}) = (\bar{\delta} + 4.383\times10^{-2})^5$$

(6) Frequency characteristics of control system

From the above, substituting Eqs. (14-2), (14-3), (14-5), (14-6), and (16-3) in Eqs. (6-1) to (6-3) yields $C_y(\bar{\delta})$ as follows:

$$C_y(\bar{\delta}) = N_C(\bar{\delta})/D_C(\bar{\delta}) \qquad (17\text{-}1)$$

where

$$N_C(\bar{\delta}) = \frac{n_{NC}(\bar{\delta})}{d_C(\bar{\delta})} \qquad (17\text{-}2)$$

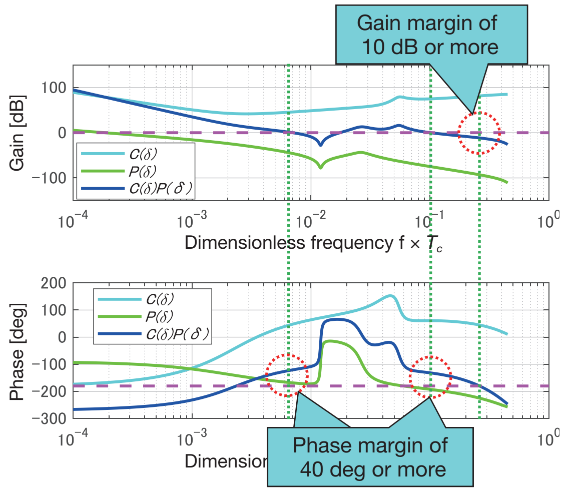
$$D_C(\bar{\delta}) = \frac{n_{DC}(\bar{\delta})}{d_C(\bar{\delta})} \qquad (17\text{-}3)$$

$$n_{NC}(\bar{\delta}) = 8500.8(\bar{\delta} + 1.381\times10^{-2}) \times (\bar{\delta}^2 + 0.2211\times10^{-2}\bar{\delta} + 1.974\times10^{-4}) \times (\bar{\delta}^2 + 0.2360\bar{\delta} + 6.286\times10^{-2})$$

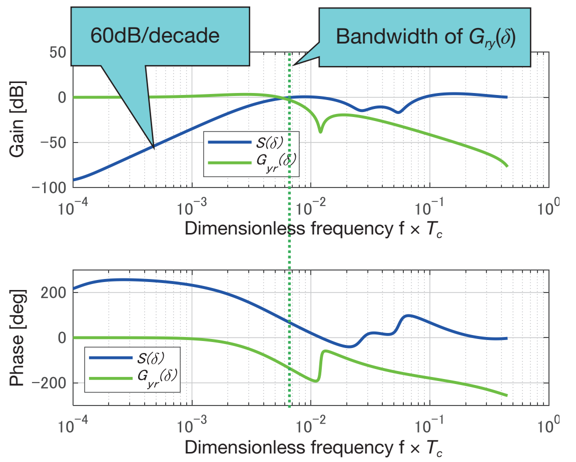
$$n_{DC}(\bar{\delta}) = \bar{\delta}^2(\bar{\delta} + 1.107) \times (\bar{\delta}^2 + 0.1766\bar{\delta} + 0.1137)$$

$$d_C(\bar{\delta}) = (\bar{\delta} + 0.2364\times10^{-2})^2 \times (\bar{\delta} + 0.9930\times10^{-2})(\bar{\delta} + 0.3369)^2$$

Next, the frequency characteristics of $C_y(\bar{\delta})$, $C_y(\bar{\delta})P(\bar{\delta})$ (loop transfer function), $S(\bar{\delta})$, and $G_{yr}(\bar{\delta})$ are shown in Fig. 16.



(a) Frequency characteristics of $C_y(\delta)$, $P(\delta)$, and $C_y(\delta)P(\delta)$



(b) Frequency characteristics of transfer functions $S(\delta)$ and $G_{ry}(\delta)$

Fig. 16 Frequency characteristics of steering angle control system

As a result of the design, the blue solid line $C_y(\bar{\delta})P(\bar{\delta})$ in diagram (a) shows a gain margin of 10 dB or more and a phase margin of 40 degrees or more. It is generally said that the desirable gain margin for a servo system is 10 dB or more and the desirable phase margin is 40 degrees or more. In the dimensionless frequency range below the frequency (bandwidth) at which $G_{ry}(\bar{\delta})$ indicated by the green solid line shows a gain of approx. -3 dB in diagram (b), $S(\bar{\delta})$ indicated by the blue solid line shows low-frequency cutoff characteristics of 60 dB/decade. The control system is thus expected to deliver robustness against disturbances or fluctuations of the controlled object in the low-frequency range.

(7) Verifying the basic design

The steering angle control system in Fig. 14 was modeled on MATLAB®/Simulink® and subjected to simulation by setting $P(\bar{\delta})$ of Eq. (14-1), $Fc(\bar{\delta})$ of Eq. (16-6), $N_c(\bar{\delta})$ and $D_c(\bar{\delta})$ of Eqs. (14-2) and (14-3), and $G_r(\bar{\delta})$ of Eq. (15), as well as parameters of the disturbance generator $G_d(\bar{\delta})$. Transfer Fcn of the Continuous library was used as a transfer function model. Setting the solver to a fixed step ode1 (Euler) and the fixed step size to one (1) allowed computation of a dimensionless discrete time system using the δ operator. Major parameters that were set for the simulation are shown in Table 1.

The results of the simulation are shown in Fig. 17. According to Fig. 17, the control output $y(k)$ follows asymptotically to the target steering angle $r(k)$ that changes in ramp rate until the sampling count k is 500. When a ramp disturbance $d(k)$ is applied at $k = 1000$, the control output $y(k)$ almost does not change and continues following the target steering angle $r(k)$. No vibration can be found with the steering wheel angle θ_h (k) as well.

Table 1 Major parameter settings for simulation

Parameter	Settings/Description
Target steering angle signal source r_0	1/500 (0 at $k = 500$)
Disturbance signal source d_0	1/500 (applied at $k = 1000$)
Disturbance generator	$G_d(\bar{\delta}) = P(\bar{\delta})/(\bar{\delta})^2$
Solver	Fixed step ode1 (Euler)
Fixed step size	1 (dimensionless time)

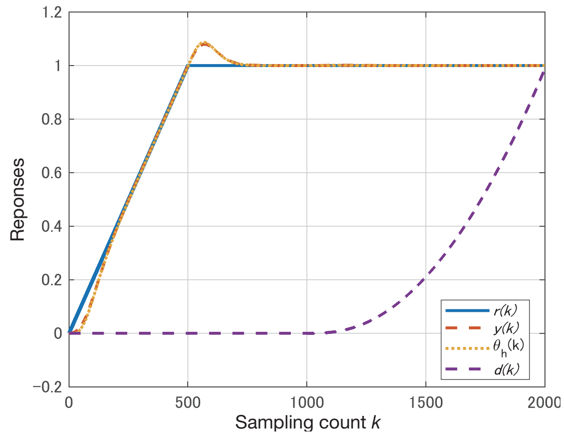


Fig. 17 Results of simulation of steering angle control system

4.3 Detailed Design and Implementation of Compensators

4.3.1 Detailed Design of Compensators

This section describes the detailed design of the compensators. Fig. 18 shows a block diagram of a compensator implementation model. In Fig. 18, the six transfer functions represented by the aforementioned Transfer Fcn are collectively called a "function model" and the transfer function of such a function model implemented in an appropriate way is called an "implementation model" (the term "implement" here refers to expressing a transfer function using state and output equations). For the steering assist control compensator, the pre-compensator in the Figure is $F_c(\delta) = 0$. In $R(\delta)$, its numerator polynomial parameters depend on the magnitude of the steering assist (assist gain), although a detailed description is omitted here. The area enclosed by the blue broken line in Fig. 18 indicates a block with state variable estimation and feedback functions and that enclosed by the red broken line is a block with disturbance estimation and feedback functions. This implies that the parametrization of a stabilizing compensator can be basically applied with the same theory as for compensators using the state variable and disturbance estimation observer [Note 4)].

Note 4) An instrument that uses a controlled object model to estimate its state variable and disturbances

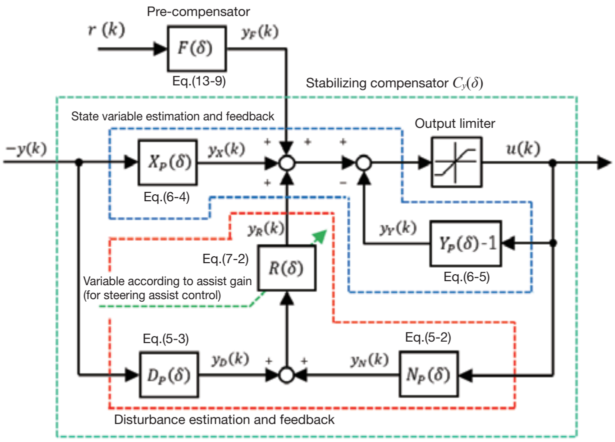


Fig. 18 Block diagram of compensator implementation model

The following introduces an example of implementation of the pre-compensator $F_c(\delta)$. The transfer function of Eq. (13-9) can be expressed as follows:

$$F_C(\delta) = n_F(\delta)/d_F(\delta) + D_F \qquad (18\text{-}1)$$

where

$$n_F(\delta) = n_{Fn+l-2}\delta^{n+l-2} + \cdots n_{F1}\delta + n_{F0}$$

$$d_F(\delta) = \delta^{n+l-1} + d_{Fn+l-2}\delta^{n+l-2} + \cdots d_{F1}\delta + d_{F0}$$

D_F: constant representing a feedthrough term of $F_C(\delta)$

A possible realization method for lower computation error may be balanced realization (calculation using the MATLAB® function "balreal" for instance). To reduce the computation complexity, we decided to select controllable canonical form (this calculation can also be achieved with the MATLAB® function "canon").

The controllable canonical form of Eq. (18-1) can be expressed by the equation below:

$$\delta x_F(k) = A_F x_F(k) + B_F r(k) \qquad (18\text{-}2)$$

$$y_F(k) = C_F x_F(k) + D_F r(k) \qquad (18\text{-}3)$$

where

$$x_F(k) = [x_{F1}(k) \quad x_{F2}(k) \quad \cdots \quad x_{Fn+l-1}(k)]^T$$

$$A_F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 1 \\ -d_{F0} & -d_{F1} & -d_{F2} & & -d_{Fn+l-2} \end{bmatrix}$$

$$B_F = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C_F = [n_{F0} \quad n_{F1} \quad \cdots \quad n_{Fn+l-2}]$$

Eq. (18-2) can be developed into the equation below:

$$\left.\begin{array}{l} x_{F1}(k) = x_{F1}(k-1) + T_c x_{F2}(k-1) \\ \quad \vdots \\ x_{Fn+l-2}(k) = x_{Fn+l-2}(k-1) \\ \qquad\qquad + T_c x_{Fn+l-1}(k-1) \\ x_{Fn+l-1}(k) = x_{Fn+l-1}(k-1) \\ \quad - T_c\left[\sum_{i=1}^{n+l-1}\{d_{Fi-1}x_{Fi}(k-1)\} + r(k-1)\right] \end{array}\right\} \qquad (18\text{-}4)$$

A compensator implementation model is created on MATLAB®/Simulink® in such a manner that computation is carried out in the order of Eq. (18-4) and Eq. (18-3). For higher model readability, the model creation should be conducted according to the modeling guideline in which the description rule was established based on MAAB [Note 5), 7)].

Note 5) An acronym of MathWorks® Automotive Advisory Board. A guideline that specifies protocols for MathWorks products including description rules.

Compensator computation by microprocessors uses the single precision floating-point format for memory saving. Therefore, control parameters (variables having a fixed value) in Eqs. (18-3) and (18-4) and state variables (whose value changes with time) are all defined with single precision floating-point variables in the implementation model stage.

4.3.2 Verifying the Compensator Implementation Model

The function and implementation models consisting of the six transfer functions in Fig. 18 are each subject to a back-to-back test to verify that these models are equivalent to each other.

4.3.3 Implementing Compensators

As described in section 3.3, Embedded Coder® is used to automatically generate a C code from the implementation model. The generated C code is checked for conformance to the MISRA-C® [Note 6)] rules using a static analysis tool. Any nonconforming items are remedied [7)]. Verification of the implementation results will be described in the next chapter.

Note 6) Coding standards intended to ensure safe, portable, and reliable software (in C-language)

5 Design Verification

This chapter provides an example of the results of verification of the compensator design for steering assist control according to the verification process.

5.1 Compensator Unit Evaluation

Software integrated with a designed compensator was implemented in the power pack. The power pack was then applied with sinusoidal signals as motor current commands. From input and output signals of the compensator, the frequency characteristics of the compensator $C_y(\delta)$ were measured. The results are shown in Fig. 19.

The measurement results closely match the functional model, verifying that the compensator has been implemented as designed. Note that the results shown in Fig. 19 are for another design different from the basic design described in 4.1.5 Example of Basic Design of Compensator.

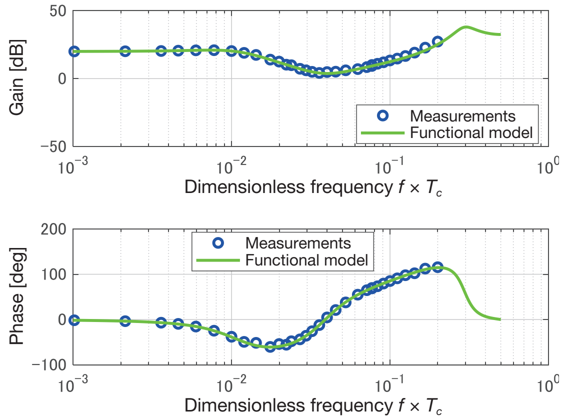


Fig. 19 Example of results of unit evaluation of compensator $C_y(\delta)$

5.2 System Evaluation and Redesigning of Compensators

The system shown in Fig. 8 was applied with sinusoidal waves as motor current commands to measure the frequency characteristics of the controlled object. The frequency characteristics of the controlled object are shown in Fig. 20. The figure plots the frequency characteristics of the compensator design model of Eq. (8-1) (hereinafter the "basic design model"), of the measurement results, and of a model identified from the measurement results.

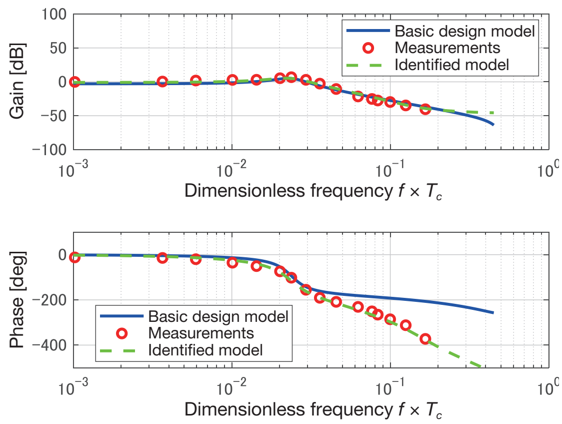


Fig. 20 Frequency characteristics of controlled object

The identified model was obtained by calculating the frequency response vector from the measurement data using the MATLAB® function "fft" and then determined with the MATLAB® function "invfreqz". Transfer function parameters determined with "invfreqz" are written in the z operator form. The obtained parameters were thus converted into the δ operator form. The identified model in Fig. 20 can be expressed by the equation below:

$$P(\bar{\delta}) = \frac{n_P(\bar{\delta})}{d_P(\bar{\delta})} = \frac{n_{P3}\bar{\delta}^3 + n_{P2}\bar{\delta}^2 + n_{P1}\bar{\delta} + n_{P0}}{\bar{\delta}^4 + d_{P3}\bar{\delta}^3 + d_{P2}\bar{\delta}^2 + d_{P1}\bar{\delta} + d_{P0}} \quad (19\text{-}1)$$

where

$$n_P(\bar{\delta}) = 5.317\times10^{-3}(\bar{\delta}+0.1864) \times(\bar{\delta}^2-0.2701\bar{\delta}+1.600)$$

$$d_P(\bar{\delta}) = (\bar{\delta}^2+0.5513\bar{\delta}+8.201\times10^{-2}) \times(\bar{\delta}^2+9.755\times10^{-2}\bar{\delta}+2.135\times10^{-2})$$

All the gains of the basic design model, measurements, and identified model decrease after around a dimensionless frequency of 0.02 at nearly -40 dB/decade, showing relatively good agreement with each other. However, the phase of the measurements is much behind the basic design model after around a dimensionless frequency of 0.03. Possible causes include a response delay of the actual current from the motor current command, the control computation time of the software, frictional torque and small rattling of the machinery, and deflection of the elements that were assumed to be rigid bodies, which were not taken into account in Eq. (8-1).

Since the gain and phase of the measurements are in good agreement with those of the identified model, the controlled object can be approximately expressed in a transfer function of degree 4 as shown in Eq. (19-1).

In fact, the frequency characteristics of the actual controlled object show a larger phase delay than that of the basic design model. This means that a compensator designed based on the basic design model may fail to keep the control system stable. Then, we have redesigned the compensator using the identified model of Eq. (19-1). The redesigned compensator can be expressed by the equation below:

$$C_y(\bar{\delta}) = \frac{n_{NC}(\bar{\delta})}{n_{DC}(\bar{\delta})} \quad (19\text{-}2)$$

where

① For large steering assist torque:

$$n_{NC}(\bar{\delta}) = 22.847(\bar{\delta}+8.494\times10^{-2}) \times(\bar{\delta}^2+0.5513\bar{\delta}+8.201\times10^{-2}) \times(\bar{\delta}^2+0.1579\bar{\delta}+1.889\times10^{-2})$$

$$n_{DC}(\bar{\delta}) = (\bar{\delta}+0.1864)(\bar{\delta}+3.666\times10^{-2})^2 \times(\bar{\delta}^2+1.295\bar{\delta}+0.7986)$$

② For middle steering assist torque:

$$n_{NC}(\bar{\delta}) = 18.246(\bar{\delta}+0.1022) \times(\bar{\delta}^2+0.5513\bar{\delta}+8.201\times10^{-2}) \times(\bar{\delta}^2+0.1646\bar{\delta}+1.701\times10^{-2})$$

$$n_{DC}(\bar{\delta}) = (\bar{\delta}+0.1864)(\bar{\delta}+6.502\times10^{-2})^2 \times(\bar{\delta}^2+1.262\bar{\delta}+0.7211)$$

③ For small steering assist torque:

$$n_{NC}(\bar{\delta}) = 12.111(\bar{\delta}+0.1336) \times(\bar{\delta}^2+0.1745\bar{\delta}+1.250\times10^{-2})$$

$$\times(\bar{\delta}^2 + 0.5513\bar{\delta} + 8.201\times10^{-2})$$

$$n_{DC}(\bar{\delta}) = (\bar{\delta} + 0.1864)(\bar{\delta} + 0.1127)^2$$

$$\times(\bar{\delta}^2 + 1.200\bar{\delta} + 0.6005)$$

$1/f(\delta)$ in Eqs. (5-2) and (5-3) is the matched pole-zero model of the following stable transfer function $1/f(s)$ that has been made dimensionless with the control frequency $1/T_c$:

$$1/f(s) = 1/\ \{f_1(s)f_2(s)\} \quad (19\text{-}3)$$

where

$$f_1(s) = s^2 + 2\zeta_{f1}\omega_{f1}s + \omega_{f1}{}^2$$

$$f_2(s) = s^2 + 2\zeta_{f2}\omega_{f1}s + \omega_{f2}{}^2$$

$$0 < \omega_{f1},\quad 0 < \omega_{f2},\quad 0 < \zeta_{f1},\quad 0 < \zeta_{f2}$$

where $\zeta_{f1} = \zeta_{f2} = 1$ and $\omega_{f1} = \omega_{f2} = 2\sqrt{K_{pe}/I_{pe}}$. $1/g(\bar{\delta})$ in Eqs. (6-4) and (6-5) is the matched pole-zero model of the following stable transfer function $1/g(s)$ that has been made dimensionless with the control frequency $1/T_c$:

$$1/g(s) = 1/\ \{g_1(s)g_2(s)\} \quad (19\text{-}4)$$

where

$$g_1(s) = s + \omega_{g1},\quad g_2(s) = s^2 + 2\zeta_{g2}\omega_{g1}s + \omega_{g2}{}^2$$

where $\zeta_{g2} = 1$, $\omega_{g1} = \omega_{g2} = 2\sqrt{K_{TB}/I_{pe}}$.

Next, the parameter for the pole $1/d_d(\bar{\delta})$ of the disturbance model of Eq. (9-1) was set as follows:

$\omega_d = 0.5\sqrt{K_{pe}/I_{pe}}$: Large steering assist torque

$\omega_d = 0.9\sqrt{K_{pe}/I_{pe}}$: Medium steering assist torque

$\omega_d = 1.5\sqrt{K_{pe}/I_{pe}}$: Small steering assist torque

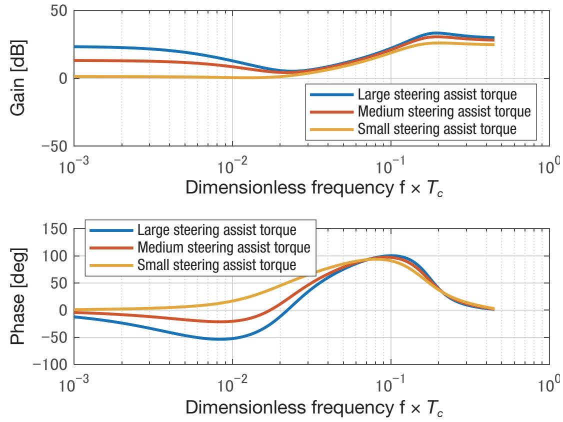
In all these cases, $\zeta_d = 1$.

Also, it is assumed that the pole of the free parameter $R(\bar{\delta})$ in Eq. (9-3) is the matched pole-zero model of $1/g_1(s)$ stated above.

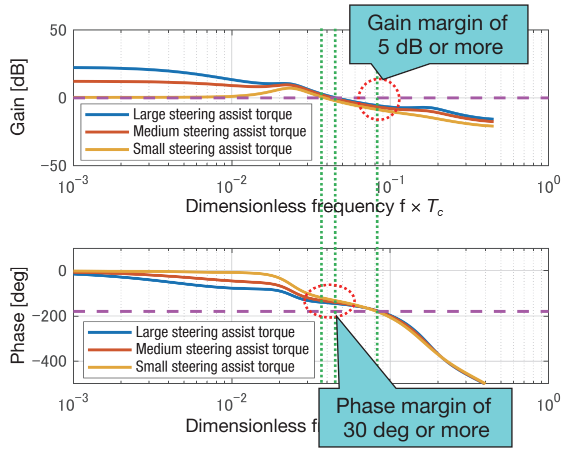
The frequency characteristics of the redesigned $C_y(\delta)$, $C_y(\delta)P(\delta)$ (loop transfer function), and $S(\delta)$ are shown in Fig. 21. As a result, the gain margin is 5 dB or more and the phase margin is 30 degrees or more in all cases. It has been verified through system evaluation and actual vehicle evaluation that a stable control system has been obtained with the redesigned compensator, although a detailed description is omitted here.

5.3 Actual Vehicle Evaluation

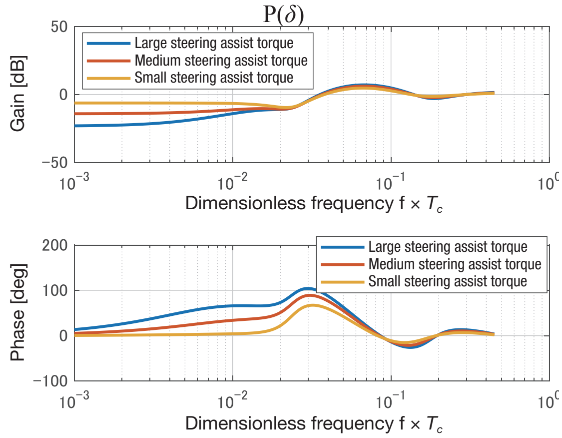
In in-house actual vehicle evaluation, a vehicle of an existing model is used to mainly test stationary steering (steering of a stopped vehicle). The test driver should start to turn the steering wheel and then turn it back to check the effect of the steering assist. The driver should also check that they feel no vibration via the wheel. The tuning parameters include the parameters of Eqs. (19-3) and (19-4), the parameters of the pole of the disturbance model of Eq. (9-1), and the parameters of the poles of Eq. (9-3). In in-house actual vehicle evaluation, several compensator candidates of different designs should be prepared by modifying the tuning parameters stated above toward actual vehicle evaluation at customer sites.



(a) Frequency characteristics of compensator Cy(δ)



(b) Frequency characteristics of loop transfer function Cy(δ) P(δ)



(c) Frequency characteristics of transfer function $S(\delta)$

Fig. 21 Frequency characteristics of redesigned steering assist control system

In actual vehicle evaluation at customer sites, a driving test is carried out on a dedicated off-road test course, in addition to the stationary steering evaluation. During the driving test, particularly the driver's feeling of steering at the start of cornering and of kickback during driving straight ahead or cornering should be checked.

6 Future Outlook

The approach explained in this report is just a compensator design method applicable to existing EPS that is a little bit different from the traditional approach. The importance of electronically-controlled in-vehicle equipment will rise as in-vehicle equipment automation and autonomous driving is further accelerated. As an in-vehicle equipment manufacturer, KYB needs to develop various value-added products. In the course of the development, not only electrical and machine element engineers but also control technology engineers should play two major roles of:

① crystallizing and screening product ideas during the product planning stage to create new products, and;

② developing a robust design to minimize uncertainty throughout the product lifecycle.

For role ①, just think about sensors, for instance. What are the minimum sensors that can deliver the functions and control performance required by the product (for cost reduction)? What physical amounts should be detected and controlled to satisfy the requirements (for higher added-value)? An effective means to solve these possible challenges is to identify controllability and observability of the system from the control technology perspective and to utilize the observer theory. For role ②, a parametric model involving uncertainty may be identified through simulation using a model with product quality (production variations) and secular changes in quality taken into account or through their testing on an actual machine. Such a parametric model can be effectively used to design compensators, ensuring robust stability of the control system.

Based on the above, we would like to strengthen our product design and proposal capabilities.

7 Concluding Remarks

Focusing on EPS for all-terrain and utility task vehicles, this report explains KYB's control technologies for in-vehicle electric actuators by introducing some numeric examples. The basis of the technologies is a design approach based on parametrization of stabilizing compensators, which has been developed into a discrete time system in the δ operator form. The various equations used for compensator calculation were converted into software programs with m-file [Note 7)]. This allows automatic calculation of parameters for the function and implementation models of compensators just by setting the tuning parameters described in Section 5.3.

Note 7) A text file that describes programs to be executed on MATLAB®

The control technologies explained in this report can be applied not only to electric actuators but also to electric pumps, hydraulic actuators, and other various electronically-controlled components.

Although compensators only make up a small proportion of in-vehicle software-controlled components, they are an essential technology directly affecting security, safety, and comfort. We would like to continue contributing to the improvement of quality and added-value of KYB products with our system analysis and evaluation techniques, control technologies, and software technologies.

Finally, we would like to take this opportunity to sincerely thank all those concerned who have extended guidance and cooperation to us in implementing and evaluating these control technologies.

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Author

MATSUMOTO Daisuke

Joined the company in 1995.
Development Sect. No.2, Electronics Technology Dept., Engineering Headquarters, Automotive Components Operations
Special Sect. Manager, Basic Technology R&D Center, Engineering Div.
Engaged in analysis of in-vehicle actuators and development of their controls

ENOMOTO Daisuke

Joined the company in 2018.
Development Sect. No.2, Electronics Technology Dept., Engineering Headquarters, Automotive Components Operations
Engaged in analysis of in-vehicle actuators and development of their controls

KENJYO Kazunori

Joined the company in 2015.
Chief Researcher, Electronics Technology Sect., Basic Technology R&D Center, Engineering Div.
Engaged in research and development of control software for in-vehicle actuators

FUKUSHI Natsuru

Joined the company in 2017.
Chief Researcher, Electronics Technology Sect., Basic Technology R&D Center, Engineering Div.
Engaged in research and development of control software for in-vehicle actuators